Revision
IBDP MAI SL

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1 A rectangle is 2680 cm long and 1970 cm wide.

a Find the perimeter of the rectangle, giving your answer in the form $a \times 10^k$, where $1 \leqslant a < 10$ and $k \in \mathbb{Z}$.

b Find the area of the rectangle, giving your answer correct to the nearest thousand square centimetres.

Mathematical Studies SL May 2009 TZ2 Paper 1 Q1

2 Simplify $\dfrac{(3xy^2)^2}{(xy)^3}$.

3 Write $(3x^2y^{-3})^{-2}$ without brackets or negative indices.

4 Zipf's law in geography is a model for how the population of a city (P) in a country depends on the rank of that city (R), that is, whether it is the largest ($R = 1$), second largest ($R = 2$), and so on. The suggested formula is $P = kR^{-1}$.

In a particular country the second largest city has a population of 2 000 000.

a Find the value of k.

b What does the model predict to be the size of the fourth largest city?

c What does the model predict is the rank of the city with population 250 000?

5 Find the exact solution of $8^x = 2^{x+6}$.

6 Show that if $a = 3\times10^8$ and $b = 4\times10^4$ then $ab = 1.2\times10^{13}$.

7 Show that if $a = 1\times10^9$ and $b = 5\times10^{-4}$ then $\frac{a}{b} = 2\times10^{12}$.

8 Show that if $a = 3\times10^4$ and $b = 5\times10^5$ then $b - a = 4.7\times10^5$.

9 The speed of light is approximately $3\times10^8\,\text{m}\,\text{s}^{-1}$. The distance from the Sun to the Earth is $1.5\times10^{11}\,\text{m}$. Find the time taken for light from the Sun to reach the Earth.

10 Find the exact solution of $\log(x+1) = 2$.

11 Find the exact solution to $\ln(2x) = 3$.

12 Use technology to solve $e^x = 2$.

13 Use technology to solve $5\times10^x = 17$.

14 Rearrange to make x the subject of $5e^x - 1 = y$.

15 **a** Given that $2^m = 8$ and $2^n = 16$, write down the value of m and of n.

b Hence or otherwise solve $8^{2x+1} = 16^{2x-3}$.

Mathematics SL May 2015 TZ1 Paper 1 Q3

16 You are given that $(7\times10^a)\times(4\times10^b) = c\times10^d$ where $1 \leqslant c < 10$ and $d \in \mathbb{Z}$.

a Find the value of c.

b Find an expression for d in terms of a and b.

17 You are given that $(6\times10^a)\div(5\times10^b) = c\times10^d$

where $1 \leqslant c < 10$ and $d \in \mathbb{Z}$.

a Find the value of c.

b Find an expression for d in terms of a and b.

18 The Henderson–Hasselbach equation predicts that the pH of blood is given by:

$$\text{pH} = 6.1 + \log\left(\frac{[HCO_3^-]}{[H_2CO_3]}\right)$$

where $[HCO_3^-]$ is the concentration of bicarbonate ions and $[H_2CO_3]$ is the concentration of carbonic acid (created by dissolved carbon dioxide). Given that the bicarbonate ion concentration is maintained at 0.579 moles per litre, find the range of concentrations of carbonic acid that will maintain blood pH at normal levels (which are between 7.35 and 7.45).

19 In attempting to set a new record, skydiver Felix Baumgartner jumped from close to the edge of the Earth's atmosphere. His predicted speed (v) in metres per second at a time t seconds after he jumped was modelled by: $v = 1350\left(1 - e^{-0.007t}\right)$.

- **a** Find the predicted speed after one second.
- **b** Baumgartner's aim was to break the speed of sound (300 ms^{-1}). Given that he was in free fall for 600 seconds, did he reach the speed of sound? Justify your answer.

20 The Richter scale measures the strength of earthquakes. The strength (S) is given by $S = \log A$, where A is the amplitude of the wave measured on a seismograph in micrometres.

- **a** If the amplitude of the wave is 1000 micrometres, find the strength of the earthquake.
- **b** If the amplitude on the seismograph multiplies by 10, what is the effect on the strength of the earthquake?
- **c** The 1960 earthquake in Chile had a magnitude of 9.5 on the Richter scale. Find the amplitude of the seismograph reading for this earthquake.

21 Solve $3 \times 20^x = 2^{x+1}$.

22 Solve the simultaneous equations:

$$9^x \times 3^y = 1$$
$$\frac{4^x}{2^y} = 16$$

23 Solve the simultaneous equations:

$$\log(xy) = 0$$
$$\log\left(\frac{x^2}{y}\right) = 3$$

Answer:

1 a 9.3×10^3 cm b 5280000 cm^2

2 $\frac{9y}{x}$

3 $\frac{y^6}{9x^4}$

4 a 4000000 b 1000000 c 16

5 $x = 3$

9 500 s

10 $x = 99$

11 $x = 0.5e^3$

12 $x = 0.693$

13 $x = 0.531$

14 $x = \ln\left(\frac{y+1}{5}\right)$

15 a $m = 3, n = 4$ b $x = 7.5$

16 a 2.8 b $a + b + 1$

17 a 1.2 b $a - b$

18 0.0259 to 0.0326

19 a 9.42 ms^{-1} b Yes

20 a 3

b Strength increases by one.

c 3160 m (It would have had to be measured using a special damped seismograph.)

21 $x = \log\left(\frac{2}{3}\right) \approx -0.176$

22 $x = 1, y = -2$

23 $x = 10, y = 0.1$

1 Pierre invests 5000 euros in a fixed deposit that pays a nominal annual interest rate of 4.5%, compounded *monthly*, for 7 years.

a Calculate the value of Pierre's investment at the end of this time. Give your answer correct to two decimal places.

Carla has 7000 dollars to invest in a fixed deposit which is compounded *annually*. She aims to double her money after 10 years.

b Calculate the minimum annual interest rate needed for Carla to achieve her aim.

Mathematical Studies SL May 2015 TZ1 Paper 1 Q10

2 Only one of the following four sequences is arithmetic and only one of them is geometric.

$$a_n = 1, 2, 3, 5, \dots$$
$$b_n = 1, \frac{3}{2}, \frac{9}{4}, \frac{27}{8}, \dots$$
$$c_n = 1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \dots$$
$$d_n = 1, 0.95, 0.90, 0.85, \dots$$

a State which sequence is

i arithmetic,

ii geometric.

b For **another** geometric sequence $e_n = -6, -3, -\frac{3}{2}, -\frac{3}{4}, \dots$

i write down the common ratio;

ii find the **exact** value of the 10th term. Give your answer as a fraction.

Mathematical Studies SL May 2015 TZ2 Paper 1 Q9

3 In an arithmetic sequence $u_8 = 10$, $u_9 = 12$.

a Write down the value of the common difference.

b Find the first term.

c Find the sum of the first 20 terms.

4 In a geometric sequence the first term is 2 and the second term is 8.

a Find the common ratio.

b Find the fifth term.

c Find the sum of the first eight terms.

5 A company projects a loss of $100 000 in its first year of trading, but each year it will make $15 000 more than the previous year. In which year does it first expect to make a profit?

6 A car has initial value $25 000. It falls in value by $1500 each year. How many years does it take for the value to reach $10 000?

7 One week after being planted, a sunflower is 20 cm tall and it subsequently grows by 25% each week.

a How tall is it 5 weeks after being planted?

b In which week will it first exceed 100 cm?

8 Kunal deposits 500 euros in a bank account. The bank pays a nominal annual interest rate of 3% compounded quarterly.

a Find the amount in Kunal's account after 4 years, assuming no further money is deposited. Give your answer to two decimal places.

b How long will it take until there has been a total of 100 euros paid in interest?

9 At the end of 2018 the world's population was 7.7 billion. The annual growth rate is 1.1%. If this growth rate continues

a estimate the world's population at the end of 2022

b what is the first year in which the population is predicted to exceed 9 billion?

10 The second term of an arithmetic sequence is 30. The fifth term is 90.

a Calculate

i the common difference of the sequence;

ii the first term of the sequence.

The first, second and fifth terms of this arithmetic sequence are the first three terms of a geometric sequence.

b Calculate the seventh term of the **geometric** sequence.

Mathematical Studies SL May 2015 TZ1 Paper 1 Q7

11 The sum of the first n terms of an arithmetic sequence is given by $S_n = 6n + n^2$.

a Write down the value of

i S_1;

ii S_2.

The nth term of the arithmetic sequence is given by u_n.

b Show that $u_2 = 9$.

c Find the common difference of the sequence.

d Find u_{10}.

e Find the lowest value of n for which u_n is greater than 1000.

f There is a value of n for which $u_1 + u_2 + \ldots + u_n = 1512$.
Find the value of n.

Mathematical Studies SL May 2015 TZ2 Paper 2 Q3

12 The first three terms of an arithmetic sequence are x, $2x + 4$, $5x$. Find the value of x.

13 The audience members at the first five showings of a new play are: 24, 34, 46, 55, 64.

a Justify that the sequence is approximately arithmetic.

b Assuming that the arithmetic sequence model still holds, predict the number of audience members at the sixth showing.

14 Evaluate $\sum_{r=1}^{12} \frac{6^r}{3^r}$.

15 According to a business plan, a company thinks it will sell 100 widgets in its first month trading, and 20 more widgets each month than the previous month. How long will it take to sell a total of 400 widgets?

16 Find an expression for the nth term of the geometric sequence a^2b^2, a^4b, a^6.

17 When digging a rail tunnel there is a cost of \$10 000 for the first metre. Each additional metre costs \$500 dollars more per metre (so the second metre costs \$10 500). How much does it cost to dig a tunnel 200 m long?

18 When Elsa was born, her grandparents deposited \$100 in her savings account and then on subsequent birthdays they deposit \$150 then \$200 then \$250 and so on in an arithmetic progression. How much have they deposited in total just after her 18th birthday?

19 £5000 is invested for 3 years in an account paying 5.8% annual interest. Over the same period, inflation is judged to be 2.92% annually. What is the real terms percentage increase in value of the investment at the end of the 3 years?

20 A company purchases a computer for \$2000. It assumes that it will depreciate in value at a rate of 10% annually. If inflation is predicted to be 2% annually, what is the real terms value of the computer after 4 years?

21 Cameron invests \$1000. He has a choice of two schemes:

Scheme A offers \$25 every year.

Scheme B offers 2% interest compounded annually.

Over what periods of investment (in whole years) is scheme A better than scheme B?

22 In a game, n small pumpkins are placed 1 metre apart in a straight line. Players start 3 metres before the first pumpkin.

Each player **collects** a single pumpkin by picking it up and bringing it back to the start. The nearest pumpkin is collected first. The player then collects the next nearest pumpkin and the game continues in this way until the signal is given for the end.

Sirma runs to get each pumpkin and brings it back to the start.

a Write down the distance, a_1, in metres that she has to run in order to **collect** the first pumpkin.

b The distances she runs to **collect** each pumpkin form a sequence $a_1, a_2, a_3, \ldots$.

i Find a_2. **ii** Find a_3.

c Write down the common difference, d, of the sequence.

The final pumpkin Sirma **collected** was 24 metres from the start.

d **i** Find the total number of pumpkins that Sirma **collected**.

ii Find the total distance that Sirma ran to **collect** these pumpkins.

Peter also plays the game. When the signal is given for the end of the game he has run 940 metres.

e Calculate the total number of pumpkins that Peter **collected**.

f Calculate Peter's distance from the start when the signal is given.

Mathematical Studies SL November 2014 Paper 2 Q5

23 The seventh, third and first terms of an arithmetic sequence form the first three terms of a geometric sequence.

The arithmetic sequence has first term a and non-zero common difference d.

a Show that $d = \frac{a}{2}$.

The seventh term of the arithmetic sequence is 3. The sum of the first n terms in the arithmetic sequence exceeds the sum of the first n terms in the geometric sequence by at least 200.

b Find the least value of n for which this occurs.

24 An athlete is training for a marathon. She considers two different programs. In both programs on day 1 she runs 10 km.

In program A she runs an additional 2 km each day compared to the previous day.

In program B she runs an additional 15% each day compared to the previous day.

a In which program will she first reach 42 km in a day. On what day of the program does this occur?

b In which program will she first reach a total of 90 km run. On what day of the program does this occur?

25 A teacher starts on a salary of £25 000. Each year the teacher gets a pay rise of £1500. The teacher is employed for 30 years.

a Find their salary in their final year.

b Find the total they have earned during their teaching career.

c If the inflation rate is on average 1.5% each year, find the real value of their final salary at the end of their final year in terms of the value at the beginning of their career, giving your answer to the nearest £100.

26 The 10th term of an arithmetic sequence is two times larger than the fourth term.

Find the ratio: $\frac{u_1}{d}$.

27 If a, b, c, d are four consecutive terms of an arithmetic sequence, prove that $2(b - c)^2 = bc - ad$.

Answers:

1 a €6847.26 b 7.18%

2 a i d_n ii b_n b i $\frac{1}{2}$ ii $-\frac{3}{256}$

3 a 2 b −4 c 300

4 a 4 b 512 c 43 690

5 8

6 10

7 a 48.8 cm b 9

8 a €563.50 b 6.25 years

9 a 8.04 billion b 2033

10 a i 20 ii 10 b 7290

11 a i 7 ii 16 c 2 d 25 e $n = 498$ f $n = 36$

12 $x = 4$

13 b 74

14 8190

15 16 months

16 $a^{2n} b^{3-n}$

17 $11.95 million

18 $10 450

19 8.63%

20 $1212.27

21 22 years

22 a $a_1 = 6$
b i $a_2 = 8$ ii $a_3 = 10$
c 2
d i 22 ii 594 m
e 28
f 16 m

23 b 32

24 a B – 12th day b A – 6th day

25 a £68 500 b £1 402 500 c £43 800

26 3

1 $P(4, 1)$ and $Q(0, -5)$ are points on the coordinate plane.

a Determine the

i coordinates of M, the midpoint of P and Q

ii gradient of the line drawn through P and Q

iii gradient of the line drawn through M, perpendicular to PQ.

The perpendicular line drawn through M meets the y-axis at $R(0, k)$.

b Find k.

Mathematical Studies SL May 2007 Paper 1 Q10

2 The midpoint, M, of the line joining $A(s, 8)$ to $B(-2, t)$ has coordinates $M(2, 3)$.

a Calculate the values of s and t.

b Find the equation of the straight line perpendicular to AB, passing through the point M.

3 The straight line, L_1, has equation $2y - 3x = 11$. The point A has coordinates $(6, 0)$.

a Give a reason why L_1 **does not** pass through A.

b Find the gradient of L_1.

L_2 is a line perpendicular to L_1. The equation of L_2 is $y = mx + c$.

c Write down the value of m.

L_2 **does** pass through A.

d Find the value of c.

4 Two points have coordinates $A(3, 6)$ and $B(-1, 10)$.

a Find the gradient of the line AB.

Line l_1 passes through A and is perpendicular to AB.

b Find the equation of l_1.

c Line l_1 crosses the coordinate axes at the points P and Q. Calculate the area of the triangle OPQ, where O is the origin.

5 Two points have coordinates $P(-1, 2)$ and $Q(6, -4)$.

a Find the coordinates of the midpoint, M, of PQ.

b Calculate the length of PQ.

c Find the equation of a straight line which is perpendicular to PQ and passes through M.

6 Two points have coordinates $P(-1, 2, 5)$ and $A(6, -4, 3)$.

a Find the coordinates of the midpoint, M, of PQ.

b Calculate the length of PQ.

7 The line l_1 with equation $x + 2y = 6$ crosses the y-axis at P and the x-axis at Q.

a Find the coordinates of P and Q.

b Find the exact distance between P and Q.

c Find the point where the line $y = x$ meets l_1.

8 The line connecting points $M(3, -5)$ and $N(-1, k)$ has equation $4y + 7x = d$.

a Find the gradient of the line.

b Find the value of k.

c Find the value of d.

9 The vertices of a quadrilateral have coordinates $A(-3, 8)$, $B(2, 5)$, $C(1, 6)$ and $D(-4, 9)$.

a Show that $ABCD$ is a parallelogram.

b Show that $ABCD$ is not a rectangle.

10 Show that the triangle with vertices $(-2, 5)$, $(1, 3)$ and $(5, 9)$ is right angled.

11 The distance of the point $(-4, a, 3a)$ from the origin is $\sqrt{4160}$. Find two possible values of a.

12 The midpoint of the line joining points $A(2, p, 8)$ and $B(-6, 5, q)$ has coordinates $(-2, 3, -5)$.
- **a** Find the values of p and q.
- **b** Calculate the length of the line segment AB.

13 The cross section of a roof is modelled as an isosceles triangle. The width of the house is 8 m and the height of the roof is 6 m. Find the gradient of the side of the roof.

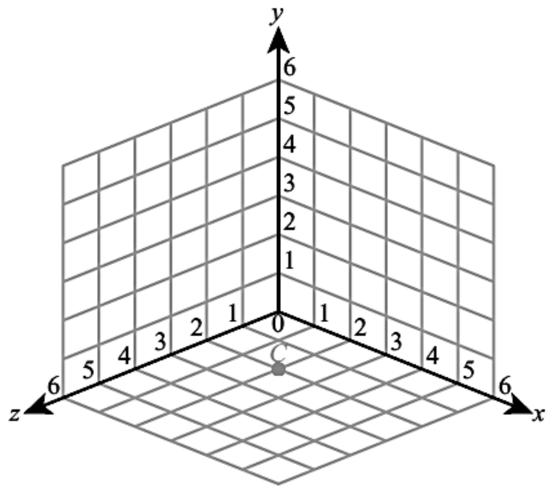
14 The lines with equations $y = \frac{1}{2}x - 3$ and $y = 2 - \frac{2}{3}x$ intersect at the point P. Find the distance of P from the origin.

15 Triangle ABC has vertices $A(-4, 3)$, $B(5, 0)$ and $C(4, 7)$.
- **a** Show that the line l_1 with equation $y = 3x$ is perpendicular to AB and passes through its midpoint.
- **b** Find the equation of the line l_2 which is perpendicular to AC and passes through its midpoint.
- **c** Let S be the intersection of the lines l_1 and l_2. Find the coordinates of S and show that it is the same distance from all three vertices.

16 Health and safety rules require that ramps for disabled access have a maximum gradient of 0.2. A straight ramp is required for accessing a platform 2 m above ground level. What is the closest distance from the platform the ramp can start?

17 The point $C(2, 0, 2)$ is plotted on the diagram on the right.
- **a** Copy the diagram and plot the points $A(5, 2, 0)$ and $B(0, 3, 4)$.
- **b** Calculate the coordinates of M, the midpoint of AB.
- **c** Calculate the length of AB.

Mathematical Studies SL November 2005 Paper 1 Q15



18 Two points have coordinates $A(-7, 2)$ and $B(5, 8)$.
- **a** Calculate the gradient of AB.
- **b** Find the coordinates of M, the midpoint of AB.
- **c** Find the equation of the line l_1 which passes through M and is perpendicular to AB. Give your answer in the form $ax + by = c$, where a, b and c are integers.
- **d** Show that the point $N(1, 1)$ lies on l_1.
- **e** Hence find the perpendicular distance from N to the line AB.

19 A straight line has equation $3x - 7y = 42$ and intersects the coordinate axes at points A and B.
- **a** Find the area of the triangle AOB, where O is the origin.
- **b** Find the length of AB.
- **c** Hence find the perpendicular distance of the line from the origin.

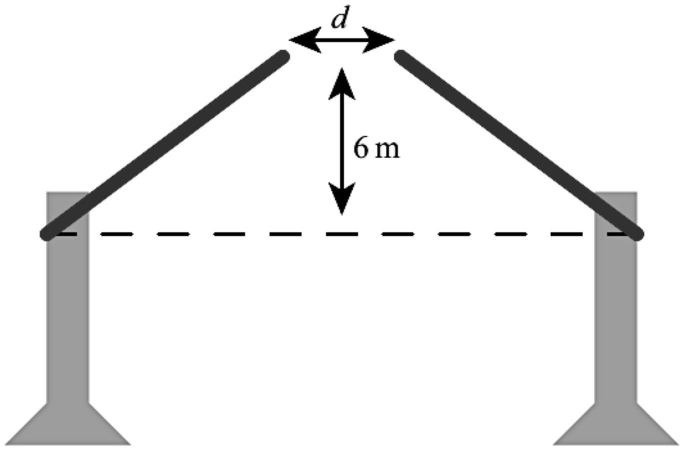
20 Line l_1 has equation $y = 5 - \frac{1}{2}x$ and crosses the x-axis at the point P. Line l_2 has equation $2x - 3y = 9$ and intersects the x-axis at the point Q. Let R be the intersection of the lines l_1 and l_2. Find the area of the triangle PQR.

21 $A(8, 1)$ and $C(2, 3)$ are opposite vertices of a square $ABCD$.
- **a** Find the equation of the diagonal BD.
- **b** Find the coordinates of B and D.

22 Quadrilateral $ABCD$ has vertices with coordinates $(-3, 2)$, $(4, 3)$, $(9, -2)$ and $(2, -3)$. Prove that $ABCD$ is a rhombus but not a square.

23 A car drives up a straight road with gradient 0.15. How far has the car travelled when it has climbed a vertical distance of 20 m?

24 A bridge consists of two straight sections, each pivoted at one end. When both sections of the bridge are raised 6 m above the horizontal, each section has gradient 0.75. Find the distance, d, between the two closest end points of the two sections.



Answer:

1 a i $(2, -2)$
ii $\frac{3}{2}$
iii $-\frac{2}{3}$
b $k = -\frac{2}{3}$

2 a $s = 6, t = -2$ b $4x + 5y = 23$

3 b $\frac{3}{2}$ c $-\frac{2}{3}$ d 4

4 a -1 b $y = x + 3$ c 4.5

5 a $(2.5, -1)$ b 9.22
c $y = \frac{7}{6}x - \frac{47}{12}$

6 a $(2.5, -1, 4)$ b 9.43

7 a $P(0, 3)$ $Q(6, 0)$ b $\sqrt{45}$
c $(2, 2)$

8 a $-\frac{7}{4}$ b $k = 2$ c $d = 1$

11 ± 20

12 a $p = 1, q = -18$ b 27.5

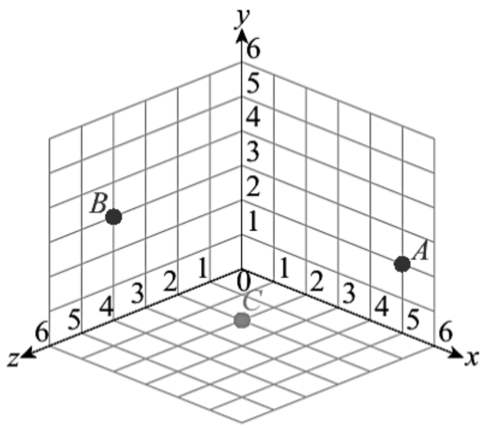
13 1.5

14 4.37

15 b $y = -2x + 5$ c $S(1, 3)$

16 10 m

17 a



b $(2.5, 2.5, 2)$ c 6.48

18 a $\frac{1}{2}$ b $(-1, 5)$
c $2x + y = 3$ e 4.47

19 a 42 b 15.2 c 5.51

20 4.32

21 a $y = 3x - 13$ b $(6, 5)$ and $(4, -1)$

23 135 m

24 4 m

1 Let $G(x) = 95e^{(-0.02x)} + 40$, for $20 \leqslant x \leqslant 200$.

a Sketch the graph of G.

b Robin and Pat are planning a wedding banquet. The cost per guest, G dollars, is modelled by the function $G(n) = 95e^{(-0.02n)} + 40$, for $20 \leqslant n \leqslant 200$, where n is the number of guests.

Calculate the **total** cost for 45 guests.

Mathematics SL May 2015 TZ1 Paper 2 Q5

2 Find the zeros of the function $f(x) = x - 3\ln x$.

3 a Find the largest possible domain of the function $g(x) = \sqrt{x+5}$.

b Solve the equation $g(x) = \dfrac{1}{x+3}$.

4 Find the coordinates of the points of intersection of the graph $y = 3 - x^2$ and $y = 2e^x$.

5 Sketch the graph of $y = \left|x^2 - 4x - 5\right|$, showing the coordinates of all axis intercepts and any maximum and minimum points.

6 a Sketch the graph of $y = \dfrac{3x-1}{x-3}$.

a Hence find the domain and range of the function $f(x) = \dfrac{3x-1}{x-3}$.

7 A speed of a car, $v\,\text{m}\,\text{s}^{-1}$, at time t seconds, is modelled by the equation $v = 18te^{-0.2t}$.

a Find the speed of the car after 1.5 seconds.

b Find two times when the speed of the car is $10\,\text{m}\,\text{s}^{-1}$.

c How long does it take for the car to reach the maximum speed?

8 The speeds of two runners are modelled by the equations: $v_1 = 8 - 6e^{-0.5}t$, $v_2 = 2 + 3x^2 - x^3$

where v is in $\text{m}\,\text{s}^{-1}$ and t is the time in seconds, with $0 \leqslant t \leqslant 2$.

a Show that both runners start at the same speed.

b After how long will they be running at the same speed again?

9 Water has a lower boiling point at higher altitudes. The relationship between the boiling point of water (T) and the height above sea level (h) can be described by the model $T = -0.0034h + 100$ where T is measured in degrees Celsius (°C) and h is measured in **metres** from sea level.

a Write down the boiling point of water at sea level.

b Use the model to calculate the boiling point of water at a height of 1.37 km above sea level.

Water boils at the top of Mt. Everest at 70 °C.

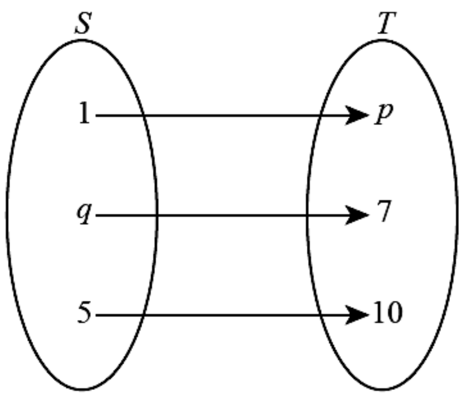
c Use the model to calculate the height above sea level of Mt. Everest.

10 To convert temperature in degrees Celsius, x to degrees Fahrenheit, f(x) a website says to multiply by 1.8 then add 32.

a Find an expression for f(x) in terms of x.

b What is the interpretation of $f^{-1}(x)$ in this context?

11 a $f : x \rightarrow 3x - 5$ is a mapping from the set S to the set T as shown on the right. Find the values of p and q.



b A function g is such that $g(x) = \dfrac{3}{(x-2)^2}$.

i State the domain of the function g(x).

ii State the range of the function g(x).

iii Write down the equation of the vertical asymptote.

12 A function is defined on the domain $x \geqslant 7$ by the equation $f(x) = 3x - 1$.

a Find the range of f.

b Find $f^{-1}(35)$.

13 Solve the equation $\frac{2x-1}{x+4} = 4 - x$.

14 Find all the roots of the equation $|x - 2| = \frac{1}{x}$.

15 Find the domain and range of the the function $h(x) = \frac{18}{x^2 - 9}$.

16 Find the range of the function $f(x) = x^2 - 7x + 3$ defined on the domain $0 < x < 6$.

17 A function is defined by $f(x) = 5e^x - 4x$.

a Find the smallest possible value of $f(x)$.

b Solve the equation $f^{-1}(x) = 2$.

18 The function $g(x) = 8 - 3x^2$ is defined on the domain $-3 < x < 2$.

a Find the range of the function.

b Solve the equation $g(x) = 5$.

c Explain why the equation $g(x) = -20$ has no solutions.

19 Sketch a possible graph of a function which is defined for all real values of x, has a horizontal asymptote $y = 5$, and crosses the coordinate axes at (0, 3) and (–1, 0).

20 A cup of tea initially has temperature 90°C and is left to cool in a room of temperature 20°C. Sketch the graph showing how the temperature of the tea changes with time.

21 In a biology experiment, there are initially 600 bacteria. The population of bacteria increases but, due to space constraints, it can never exceed 2000. Sketch a possible graph showing how the number of bacteria changes with time.

22 A child pushes a toy car in the garden, starting from rest, and then lets it go. The speed of the car, $v\,\text{m s}^{-1}$, after t seconds is modelled by the equation $v = 3xe^{-x}$.

a Find the maximum speed of the car.

b Suggest why this is not a good model for the speed of the car after 20 seconds.

23 Solve the equation $\left|e^{2x} - \frac{1}{x+2}\right| = 2$.

Mathematics HL May 2005 TZ2 Paper 2 Q2

24 **a** Sketch the graph of $y = \frac{x-12}{\sqrt{x^2 - 4}}$.

b Write down

i the x-intercept;

ii the equations of all asymptotes.

25 Solve the equation $e^x = x^3 - 2$.

26 A function is defined by $f(x) = \frac{x^2 - 9x}{5x + 1}$ for $x > 1$. Find the minimum value of this function.

27 A function is defined by $f(x) = \frac{10x^2 + 7}{x^2 - 4x}$.

a Write down the largest possible domain of the function.

b Find the range of the function for the domain from part **a**.

28 **a** Find the largest domain of the function $g(x) = \frac{2x}{3 + \ln x}$.

b For the domain found in part **a** find the range of the function.

29 A function is defined by $g(x) = 2x + \ln(x - 2)$.

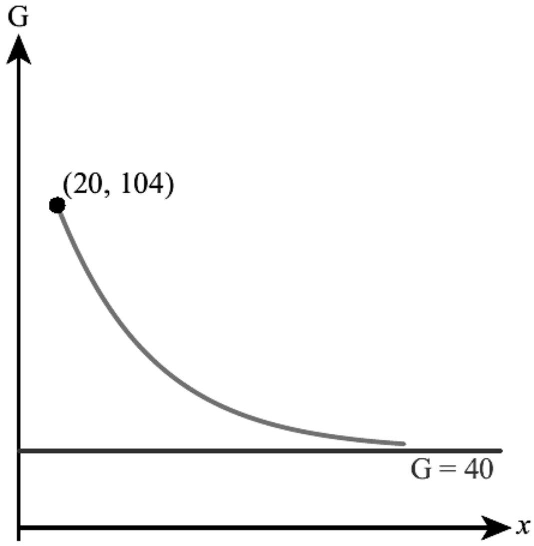
a State the domain and range of g.

b Sketch the graph of $y = g(x)$ and $y = g^{-1}(x)$ on the same set of axes.

c Solve the equation $g(x) = g^{-1}(x)$.

Answer:

1 a



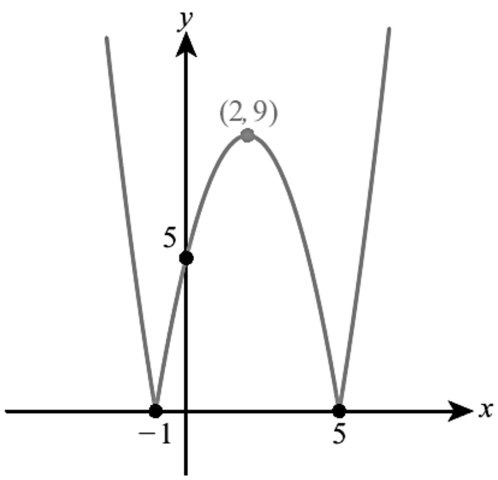
b \$3538.09

2 $x = 1.86, 4.54$

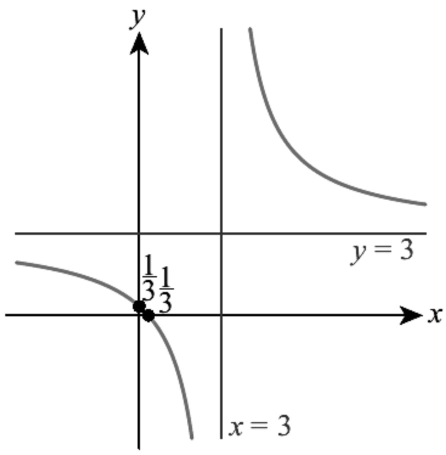
3 a $x \geqslant -5$ b $x = -2.38$

4 (–1.61, 0.399) and (0.361, 2.87)

5



6 a



b $x \neq 3$, $f(x) \neq 3$

7 a $20.0\,\mathrm{m\,s^{-1}}$ b 0.630, 17.1 s c 5 s

8 b 1.30 s

9 a 100°C b 95.3°C c 8.82 km

10 a $f(x) = 1.8x + 32$

b Temperature in Celsius if x is temperature in Fahrenheit.

7 a $20.0\,\mathrm{m\,s^{-1}}$ b 0.630, 17.1 s c 5 s

8 b 1.30 s

9 a 100°C b 95.3°C c 8.82 km

10 a $f(x) = 1.8x + 32$

b Temperature in Celsius if x is temperature in Fahrenheit.

11 a $p = -2$, $q = 4$

b i $x \neq 2$

ii $g(x) > 0$

iii $x = 2$

12 a $f(x) \geqslant 20$ b 12

13 $x = -5.24, 3.24$

14 $x = 1, 2.41$

15 $x \neq \pm 3$, $h(x) \leqslant -2$ or $h(x) > 0$

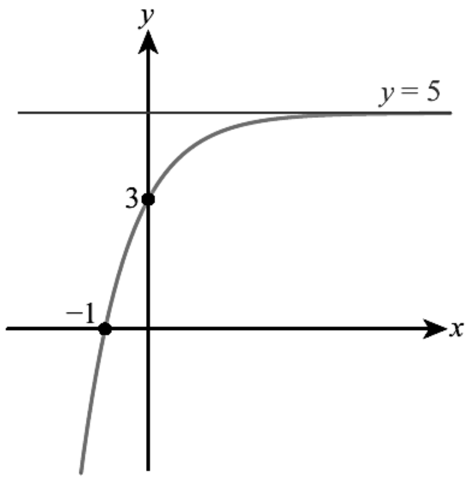
16 $-9.25 \leqslant f(x) < 3.$

17 a 4.89 b $x = 28.9$

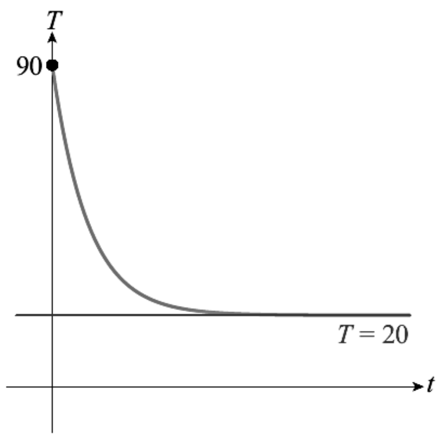
18 a $-19 < g(x) \leqslant 8$ b $x = -1$, $x = 1$

c –20 is not in the range of g

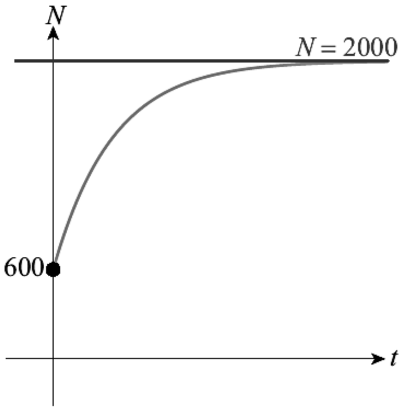
19



20



21

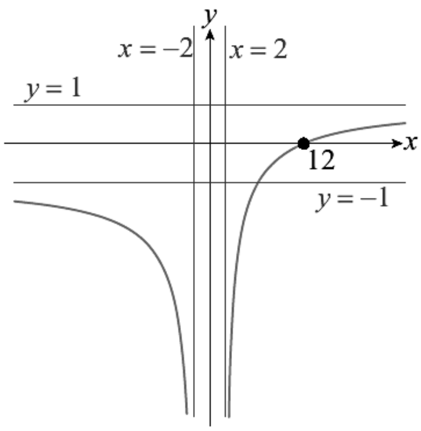


22 a $1.10\ \text{m s}^{-1}$

b e.g. The car will stop by then

23 $x = -2.50, -1.51, 0.440$

24 a



b i (12, 0)

ii $x = \pm 2, y = \pm 1$

25 $x = 2.27, 4.47$

26 -1.34

27 a $x \neq 0, 4$

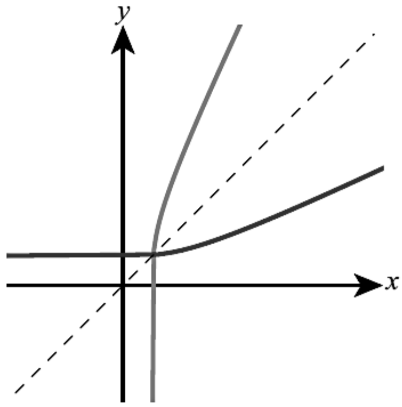
b $f(x) \leqslant -5.15$ or $f(x) \geqslant 3.40$

28 a $x > 0, x \neq e^{-3}$

b $g(x) < 0$ or $g(x) \geqslant 0.271$

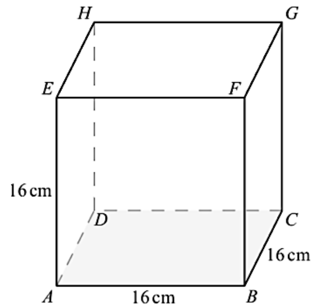
29 a $x > 2$, $g(x) \in \mathbb{R}$

b



c $x = 2.12$

1 Viewed from 50m away, a building has an angle of elevation of 35°. Find the height of the building.



2 The cube in the diagram has side 16cm.

a Find the lengths of AC and AG.

b Draw a sketch of triangle ACG, labelling the lengths of all the sides.

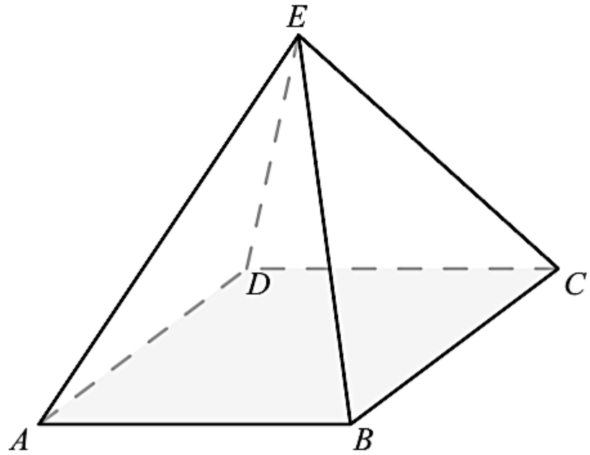
c Find the angle between AC and AG.

3 The base of a pyramid is a square of side 23cm. The angle between AC and AE is 56°.

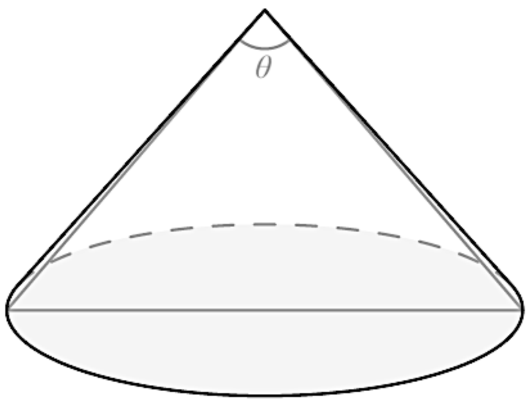
a Find the length of AC.

b Find the height of the pyramid.

c Find the length of AE.



4 A cone has radius 5cm and vertical height 12cm. Find the size of angle θ.



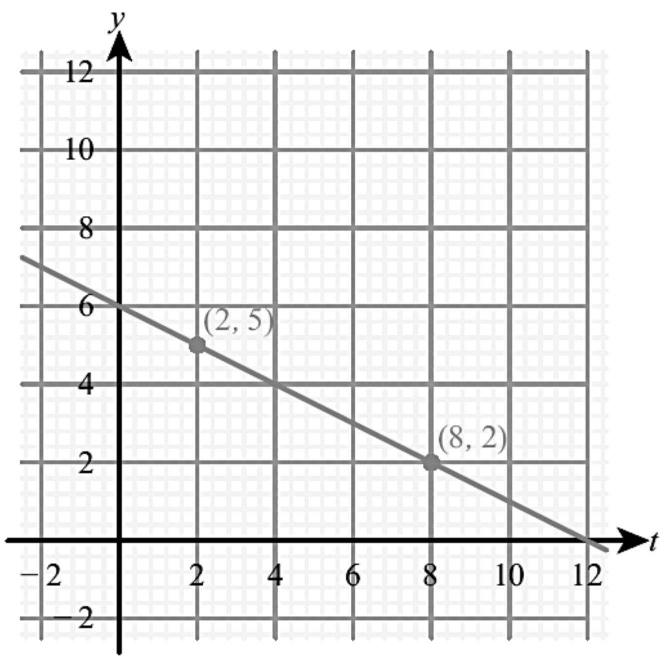
5 A and B are points on a straight line as shown on the graph on the right.

a Write down the y-intercept of the line AB.

b Calculate the gradient of the line AB.

The acute angle between the line AB and the x-axis is θ.

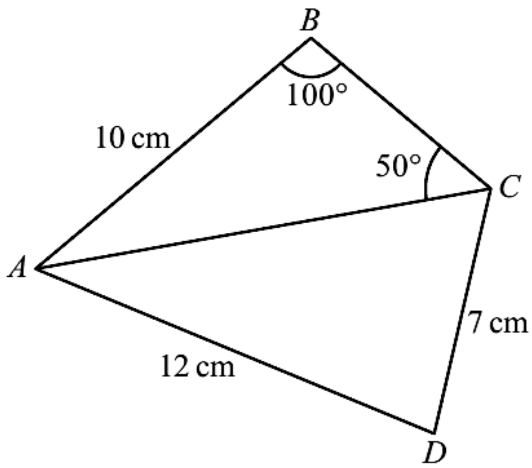
c Calculate the size of θ.



6 The quadrilateral $ABCD$ has $AB = 10$ cm. $AD = 12$ cm and $CD = 7$ cm.
The size of angle ABC is 100° and the size of angle ACB is 50°.

a Find the length of AC in centimetres.

b Find the size of angle ADC.



7 In triangle ABC, $A = 50°$, $B = 70°$ and $a = 10$ cm. Find the length of side b.

8 In triangle ABC, $A = 15°$, $b = 8$ cm and $c = 10$ cm. Find the length of side a.

9 In triangle ABC, the sides are $a = 3$ cm, $b = 5$ cm and $c = 7$ cm. Find angle A.

10 Triangle XYZ has $X = 42°$, $x = 15$ cm and $y = 12$ cm. Find angle Y.

11 Triangle PQR has $P = 120°$, $p = 9$ cm and $q = 4$ cm. Find angle R.

12 In triangle ABC, $B = 32°$, $C = 72°$ and $b = 10$ cm. Find side a.

13 After recording the angle of elevation of the top of a tower at an unknown distance from the tower's base, a student walks exactly 20 m directly away from the tower along horizontal ground and records a second angle of elevation. The two angles recorded are 47.7° and 38.2°. Find the height of the tower.

14 All that remains intact of an ancient castle is part of the keep wall and a single stone pillar some distance away. The base of the wall and the foot of the pillar are at equal elevations.

From the top of the keep wall, the tip of the pillar is at an angle of depression of 23.5° and the base of the pillar is at an angle of depression of 37.7°.

The wall is known to have a height of 41 m. Find the height of the pillar, to the nearest metre.

15 **a** Sketch the lines with equations $y = \frac{1}{3}x + 5$ and $y = 10 - x$, showing all the axis intercepts.

b Find the coordinates of the point of intersection between the two lines.

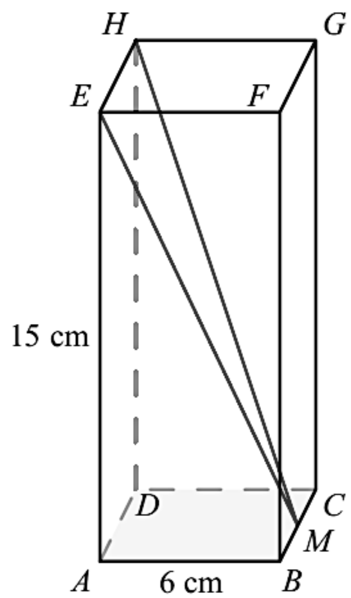
c Find the size of the acute angle between the two lines.

16 A square-based pyramid has height 26 cm. The angle between the height and one of the sloping edges is 35°. Find the volume of the pyramid.

17 The base of a cuboid $ABCDEFGH$ is a square of side 6 cm. The height of the cuboid is 15 cm. M is the midpoint of the edge BC.

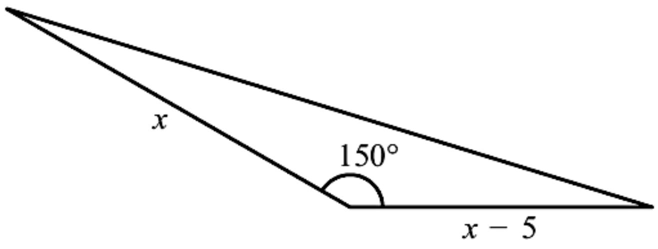
a Find the angle between ME and the base $ABCD$.

b Find the size of the angle HME.



18 In triangle ABC, $AB = x$, $AC = 2x$, $BC = x + 4$ and $B\hat{A}C = 60°$. Find the value of x.

19 The area of this triangle is 84 units². Find the value of x.



20 A 30 m tall tower and a vertical tree both stand on horizontal ground. From the top of the tower, the angle of depression of the bottom of the tree is 50°. From the bottom of the tower, the angle of elevation of the top of the tree is 35°. Find the height of the tree.

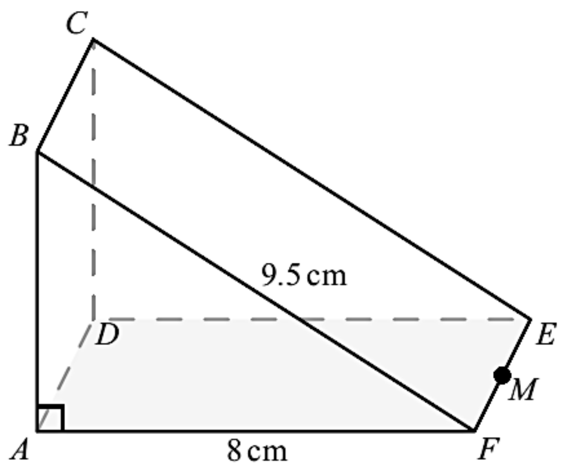
21 Tennis balls are sold in cylindrical tubes that contain four balls. The radius of each tennis ball is 3.15 cm and the radius of the tube is 3.2 cm. The length of the tube is 26 cm.

- **a** Find the volume of one tennis ball.
- **b** Calculate the volume of the empty space in the tube when four tennis balls have been placed in it.

22 The diagram shows a right triangular prism, $ABCDEF$, in which the face $ABCD$ is a square.

$AF = 8$ cm, $BF = 9.5$ cm, and angle BAF is 90°.

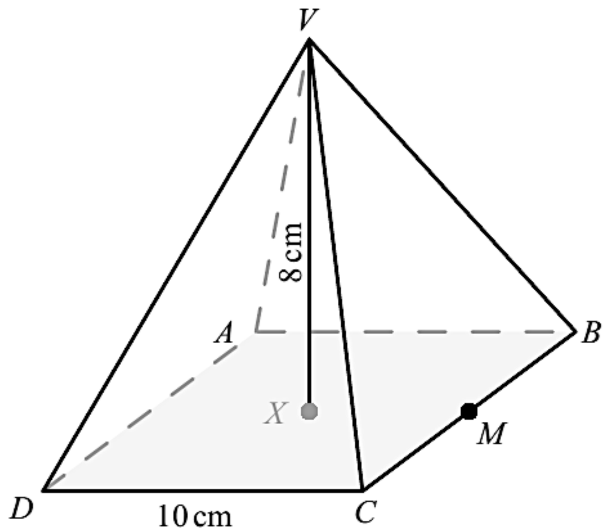
- **a** Calculate the length of AB.
  M is the midpoint of EF.
- **b** Calculate the length of BM.
- **c** Find the size of the angle between BM and the face $ADEF$.



23 **Part A**

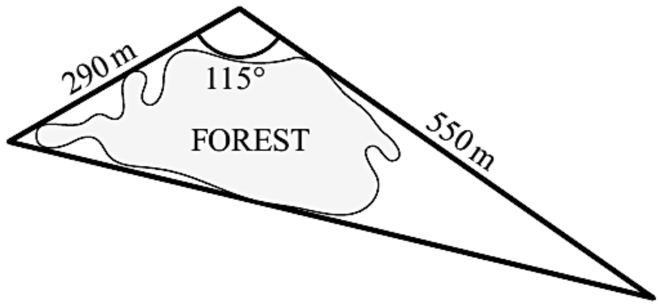
The diagram on the right shows a square-based right pyramid. $ABCD$ is a square of side 10 cm. VX is the perpendicular height of 8 cm. M is the midpoint of BC.

- **a** Write down the length of XM.
- **b** Calculate the length of VM.
- **c** Calculate the angle between VM and $ABCD$.



Part B

A path goes around a forest so that it forms the three sides of a triangle. The lengths of two sides are 550 m and 290 m. These two sides meet at an angle of 115°. A diagram is shown on the right.

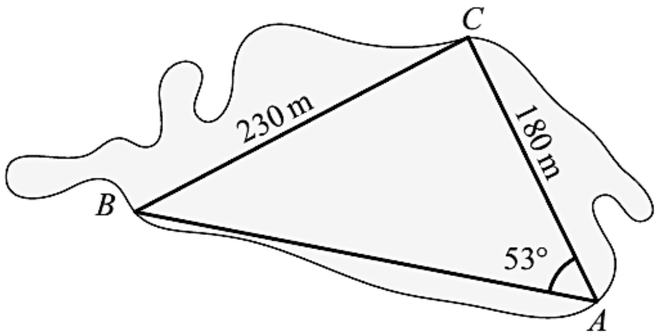


a Calculate the length of the third side of the triangle. Give your answer correct to the nearest 10 m.

b Calculate the area enclosed by the path that goes around the forest.

Inside the forest a second path forms the three sides of another triangle named *ABC*. Angle *BAC* is 53°, *AC* is 180 m and *BC* is 230 m.

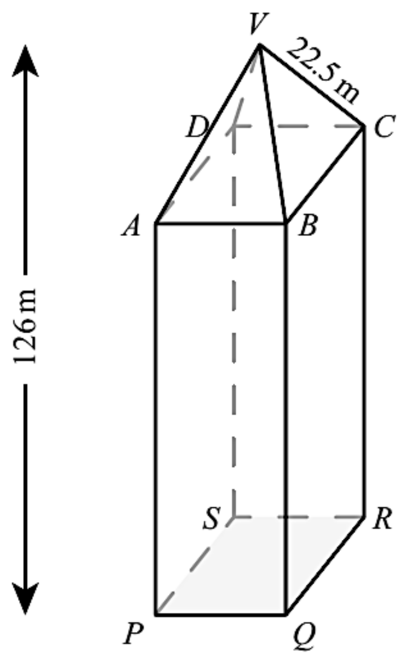
c Calculate the size of angle *ACB*.



24 The diagram shows an office tower of total height 126 metres. It consists of a square-based pyramid *VABCD* on top of a cuboid *ABCDPQRS*.

V is directly above the centre of the base of the office tower.

The length of the sloping edge *VC* is 22.5 metres and the angle that *VC* makes with the base *ABCD* (angle *VCA*) is 53.1°.



a **i** Write down the length of *VA* in metres.

ii Sketch the triangle *VCA* showing clearly the length of *VC* and the size of angle *VCA*.

b Show that the height of the pyramid is 18.0 metres correct to 3 significant figures.

c Calculate the length of *AC* in metres.

d Show that the length of *BC* is 19.1 metres correct to 3 significant figures.

e Calculate the volume of the tower.

To calculate the cost of air conditioning, engineers must estimate the weight of air in the tower. They estimate that 90% of the volume of the tower is occupied by air and they know that 1 m^3 of air weighs 1.2 kg.

f Calculate the weight of air in the tower.

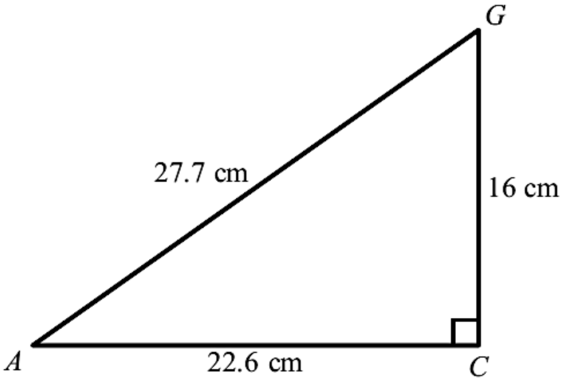
25 Find the area of the triangle formed by the lines $y = 8 - x$, $2x - y = 10$ and $11x + 2y = 25$.

Answer:

1 35.0 m

2 a $AC = 22.6$ cm, $AG = 27.7$ cm

b



c 35.3°

3 a 32.5 cm b 24.1 cm c 29.1 cm

4 45.2°

5 a 6 b $-\frac{1}{2}$ c 26.6°

6 a 12.9 cm

b 80.5°

7 12.3 cm

8 3.07 cm

9 21.8°

10 32.4°

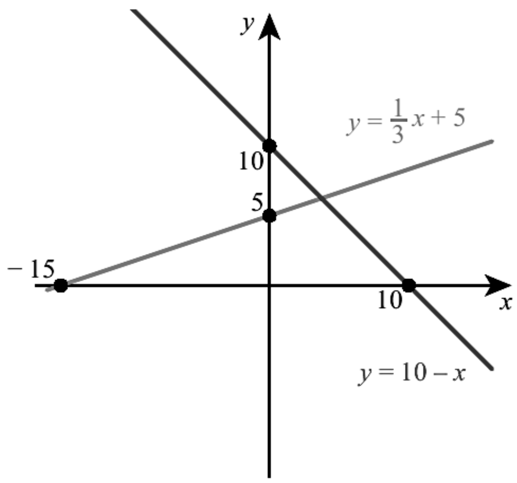
11 37.4°

12 18.3 cm

13 55.4 m

14 17.9 m

15 a



b (3.75, 6.25)

c 63.4°

16 23 900 cm^3

17 a 65.9° b 21.0°

18 5.46

19 21

20 17.6 m

21 a 131 cm^3 b 313 cm^3

22 a 5.12 cm b 9.84 cm c 31.4°

23 a 5 cm

b 9.43 cm

c 58.0°

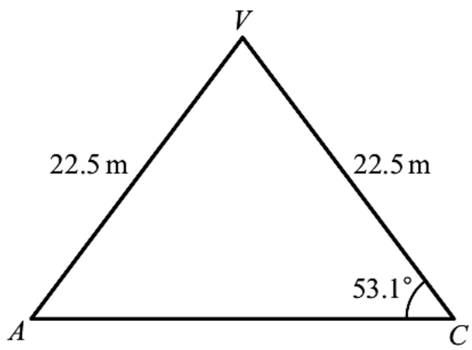
a 720 m

b 72 300 m^2

c 88.3°

24 a i 22.5 m

ii



c 27.0 m

e 41 600 m^3

f 44 900 kg

25 22.5

1 In a clinical trial, a drug is found to have a positive effect on 128 out of 200 participants. A second trial is conducted, involving 650 participants. On how many of the participants is the drug expected to have a positive effect?

2 The table shows the genders and fruit preferences of a group of children.

	Boy	Girl
Apples	16	21
Bananas	32	14
Strawberries	11	21

Find the probability that

a a randomly selected child is a girl

b a randomly selected child is a girl who prefers apples

c a randomly selected boy prefers bananas

d a randomly selected child is a girl, given that they prefer strawberries.

3 Every break time at school, Daniel randomly chooses whether to play football or basketball. The probability that he chooses football is $\frac{2}{3}$. If he plays football, the probability that he scores is $\frac{1}{5}$. If he plays basketball, the probability that he scores is $\frac{3}{4}$.

Using a tree diagram, find the probability that Daniel

a plays football and scores

b does not score.

4 The Venn diagram shows two sets:

T = {multiples of 3 between 1 and 20}

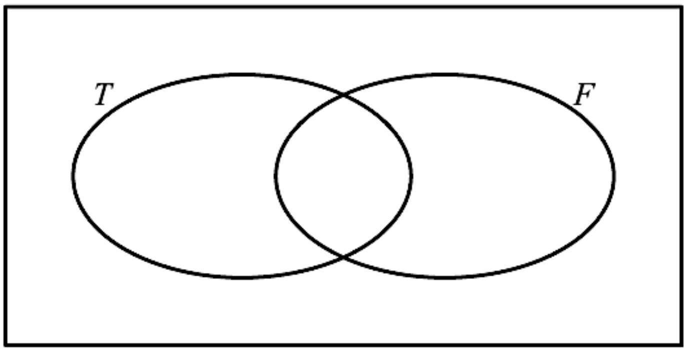
F = {multiples of 5 between 1 and 20}

a Copy the diagram and place the numbers from 1 to 20 (inclusive) in the correct region of the Venn diagram.

b Find the probability that a number is

i a multiple of 5

ii a multiple of 5 given that it is not a multiple of 3.



5 In a certain school, the probability that a student studies Geography is 0.6, the probability that a student studies French is 0.4 and the probability that they study neither subject is 0.2.

a Find the probability that a student studies both subjects.

b Given that a student does not study French, what is the probability that they study Geography?

6 Events A and B satisfy: $P(A) = 0.6$, $P(B) = 0.3$, $P(A \cup B) = 0.72$

a Find $P(A \cap B)$.

b Determine whether A and B are independent.

7 Events A and B are independent with $P(A) = 0.6$ and $P(B) = 0.8$. Find $P(A \cup B)$.

8 Events A and B satisfy: $P(A) = 0.7$, $P(B) = 0.8$ and $P(A \cap B) = 0.6$

a Draw a Venn diagram showing events A and B.

b Find $P(A \mid B)$.

c Find $P(B \mid A')$.

9 Elsa has a biased coin with the probability $\frac{1}{3}$ of showing heads, a fair six-sided dice (numbered 1 to 6) and a fair four-sided dice (numbered 1 to 4). She tosses the coin once. If it comes up heads, she rolls the six-sided dice, and if comes up tails she rolls the four-sided dice.

a Find the probability that the dice shows

i a '1'

ii a '6'.

b Find the probability that the number on the dice is a multiple of 3.

10 An integer is chosen at random from the first 10 000 positive integers. Find the probability that it is

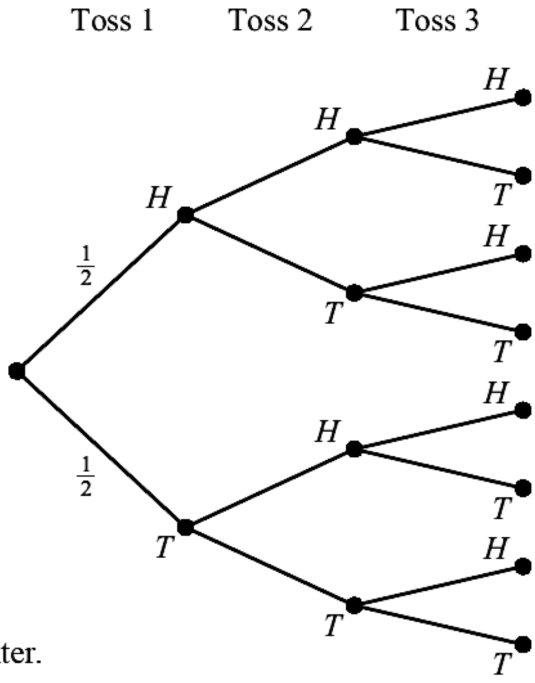
a a multiple of 7

b a multiple of 9

c a multiple of at least one of 7 and 9.

11 A fair coin is tossed three times.

a Copy and complete the tree diagram showing all possible outcomes.



Find the probability that

b the coin shows tails all three times

c the coin shows heads at least once

d the coin shows heads exactly twice.

12 There are six black and eight white counters in a box. Asher takes out two counters without replacement. Elsa takes out one counter, returns it to the box and then takes another counter. Who has the larger probability of selecting one black and one white counter?

13 A drawer contains eight red socks, six white socks and five black socks. Two socks are taken out at random. What is the probability that they are the same colour?

14 A pack of cards in a game contains eight red, six blue and ten green cards. You are dealt two cards at random.

a Given that the first card is blue, write down the probability that the second card is red.

b Find the probability that you get at least one red card.

15 Alan's laundry basket contains two green, three red and seven black socks.

He selects one sock from the laundry basket at random.

a Write down the probability that the sock is red.

Alan returns the sock to the laundry basket and selects two socks at random.

b Find the probability that the first sock he selects is green and the second sock is black.

Alan returns the socks to the laundry basket and again selects two socks at random.

c Find the probability that he selects two socks of the same colour.

16 100 students at IB College were asked whether they study Music (M), Chemistry (C) or Economics (E) with the following results.

10 study all three

15 study Music and Chemistry

17 study Music and Economics

12 study Chemistry and Economics

11 study Music **only**

6 study Chemistry **only**

a Draw a Venn diagram to represent the information above.

b Write down the number of students who study Music but not Economics.

There are 22 Economics students **in total**.

c **i** Calculate the number of students who study Economics only.

ii Find the number of students who study none of these three subjects.

A student is chosen at random from the 100 that were asked above.

d Find the probability that this student

- **i** studies Economics
- **ii** studies Music and Chemistry but not Economics
- **iii** does not study either Music or Economics
- **iv** does not study Music given that the student does not study Economics.

Mathematical Studies SL May 2013 Paper 2 TZ1 Q2

17 A bag contains 10 red sweets and n yellow sweets. Two sweets are selected at random. The probability that the two sweets are the same colour is $\frac{1}{2}$.

a Show that $n^2 - 21n + 90 = 0$.

b Hence find the possible values of n.

18 At a large school, students are required to learn at least one language, Spanish or French. It is known that 75% of the students learn Spanish, and 40% learn French.

a Find the percentage of students who learn **both** Spanish and French.

b Find the percentage of students who learn Spanish, but not French.

At this school, 52% of the students are girls, and 85% of the girls learn Spanish.

c A student is chosen at random. Let G be the event that the student is a girl, and let S be the event that the student learns Spanish.

- **i** Find $P(G \cap S)$.
- **ii** Show that G and S are **not** independent.

d A boy is chosen at random. Find the probability that he learns Spanish.

19 A group of 100 customers in a restaurant are asked which fruits they like from a choice of mangoes, bananas and kiwi fruits. The results are as follows.

15 like all three fruits

22 like mangoes and bananas

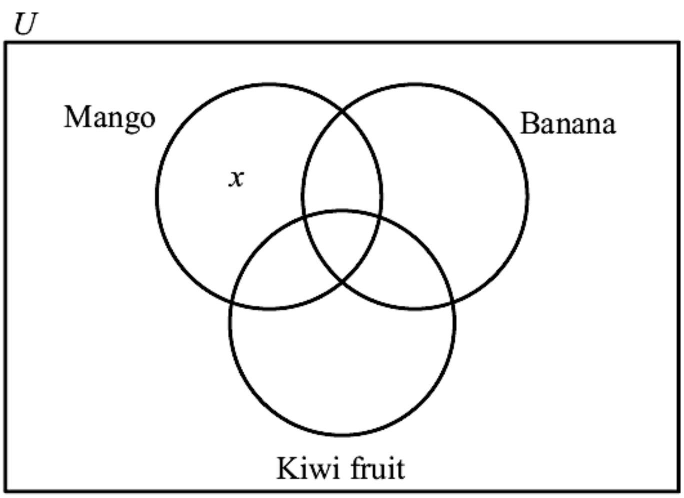
33 like mangoes and kiwi fruits

27 like bananas and kiwi fruits

8 like none of these three fruits

x like **only** mangoes

a **Copy** the following Venn diagram and correctly insert all values from the above information.



The number of customers that like **only** mangoes is equal to the number of customers that like **only** kiwi fruits. This number is half of the number of customers that like **only** bananas.

b Complete your Venn diagram from part **a** with this additional information **in terms of** x.

c Find the value of x.

d Write down the number of customers who like

- **i** mangoes;
- **ii** mangoes or bananas.

e A customer is chosen at random from the 100 customers. Find the probability that this customer

- **i** likes none of the three fruits;
- **ii** likes only two of the fruits;
- **iii** likes all three fruits given that the customer likes mangoes and bananas.

f Two customers are chosen at random from the 100 customers. Find the probability that the two customers like none of the three fruits.

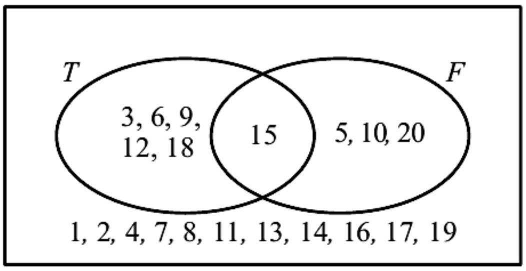
Answer:

1 416

2 a $\frac{56}{115}$ b $\frac{21}{115}$ c $\frac{32}{59}$ d $\frac{21}{32}$

3 a $\frac{2}{15}$ b $\frac{37}{60}$

4 a



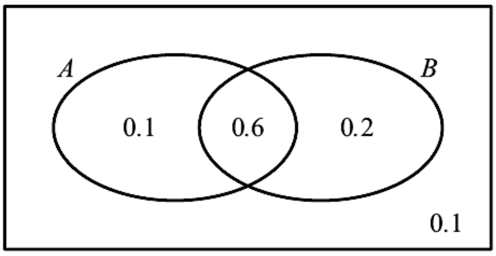
b i $\frac{1}{5}$ ii $\frac{3}{14}$

5 a 0.2 b $\frac{2}{3}$

6 a 0.18 b yes

7 0.92

8 a



b 0.75

c 0.667

9 a i $\frac{2}{9}$

b $\frac{5}{18}$

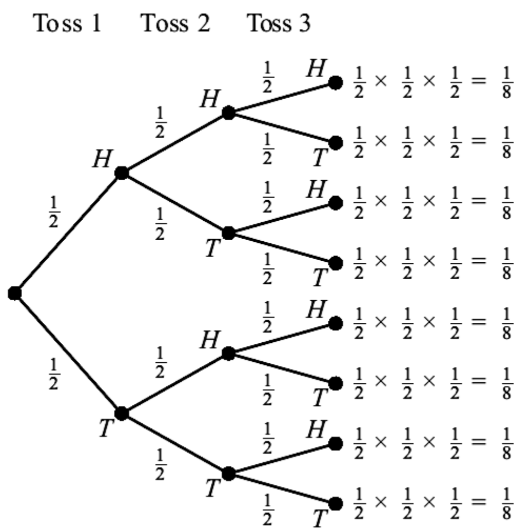
ii $\frac{1}{18}$

10 a 0.143

b 0.111

c 0.238

11 a



b $\frac{1}{8}$ c $\frac{7}{8}$ d $\frac{3}{8}$

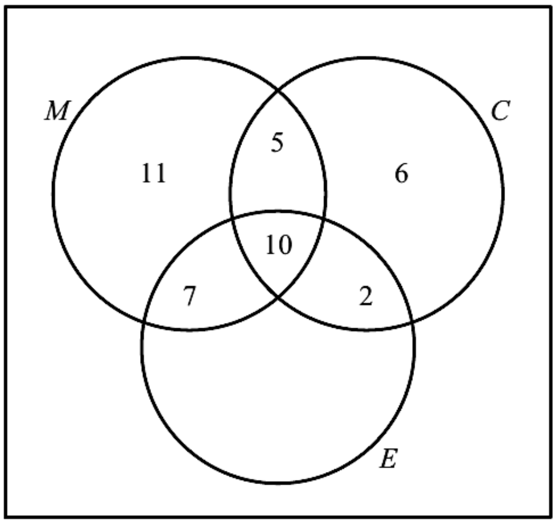
12 Asher $\left(\frac{48}{91} > \frac{24}{49}\right)$

13 0.310

14 a $\frac{8}{23}$ b $\frac{13}{23}$

15 a $\frac{1}{4}$ b $\frac{7}{66}$ c $\frac{25}{66}$

16 a



b 16

c i 3 ii 56

d i 0.22

ii 0.05

iii 0.62

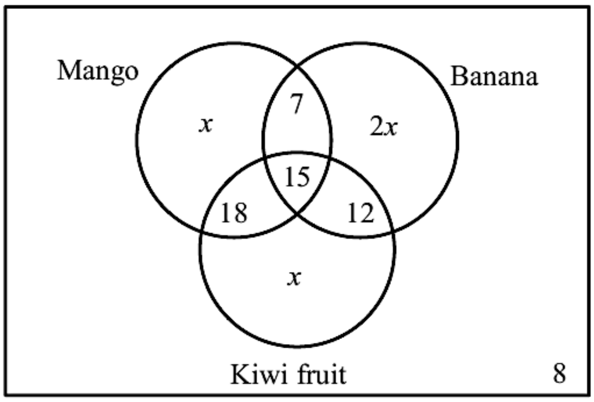
iv $\frac{31}{39}$

17 b 6 or 15

18 a 15% b 60%

c i 0.442 d 0.642

19 a, b



c $x = 10$

d i 50 ii 82

e i 0.08 ii 0.37 iii $\frac{15}{82}$

f $\frac{14}{2475}$

1 Random variable X has the probability distribution given in the table.

x	1	2	3	4
$P(X = x)$	0.2	0.2	0.1	k

a Find the value of k.
b Find $P(X \geqslant 3)$.
c Find $E(X)$.

2 A fair six-sided dice is rolled twelve times. Find the probability of getting
a exactly two 16s
b more than two 1s.

3 Lengths of films are distributed normally with mean 96 minutes and standard deviation 12 minutes. Find the probability that a randomly selected film is
a between 100 and 120 minutes long
b more than 105 minutes long.

4 Scores on a test are normally distributed with mean 150 and standard deviation 30. What score is needed to be in the top 1.5% of the population?

5 A factory making plates knows that, on average, 2.1% of its plates are defective. Find the probability that in a random sample of 20 plates, at least one is defective.

6 Daniel and Alessia play the following game. They roll a fair six-sided dice. If the dice shows an even number, Daniel gives Alessia \$1. If it shows a 1, Alessia gives Daniel \$1.50; otherwise Alessia gives Daniel \$0.50.
a Complete the table showing possible outcomes of the game for Alessia.

Outcome	\$1	–\$0.50	–\$1.50
Probability			

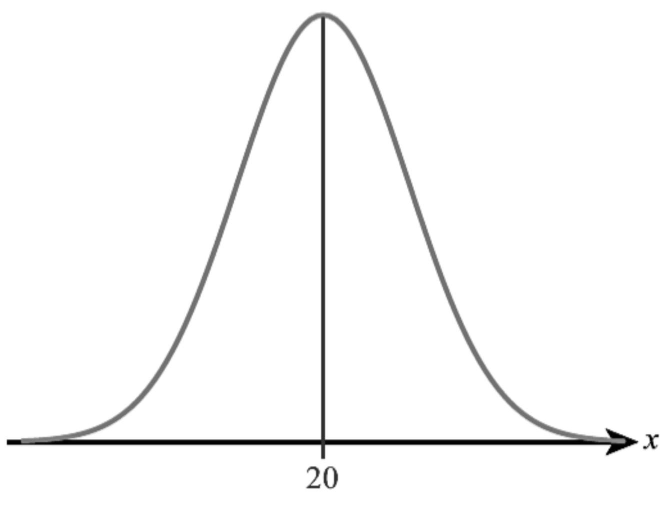
b Determine whether the game is fair.

7 A spinner with four sectors, labelled 1, 3, 6 and N (where $N > 6$) is used in a game. The probabilities of each number are shown in the table.

Number	1	3	6	N
Probability	$\frac{1}{2}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{10}$

A player pays three counters to play a game, spins the spinner once and receives the number of counters equal to the number shown on the spinner. Find the value of N so that the game is fair.

8 A random variable X is distributed normally with a mean of 20 and variance 9.
a Find $P(X \leqslant 24.5)$.
b Let $P(X \leqslant k) = 0.85$.
i Represent this information on a copy of the following diagram.



ii Find the value of k.

9 The random variable X has the distribution $B(30, p)$. Given that $E(X) = 10$, find

a the value of p;

b $P(X = 10)$;

c $P(X \geqslant 15)$.

10 The mean mass of apples is 110 g with standard deviation 12.2 g.

a What is the probability that an apple has a mass of less than 100 g?

b Apples are packed in bags of six. Find the probability that more than one apple in a bag has a mass of less than 100 g.

11 Masses of eggs are normally distributed with mean 63 g and standard deviation 6.8 g. Eggs with a mass over 73 g are classified as 'very large'.

a Find the probability that a randomly selected box of six eggs contains at least one very large egg.

b What is more likely: a box of six eggs contains exactly one very large egg, or that a box of 12 eggs contains exactly two very large eggs?

12 Bob competes in the long jump. The lengths of his jumps are normally distributed with mean 7.35 m and variance $0.64\,m^2$.

a Find the probability that Bob jumps over 7.65 m.

b In a particular competition, a jump of 7.65 m is required to qualify for the final. Bob makes three jumps. Find the probability that he will qualify for the final.

13 In an athletics club, the 100 m times of all the runners follow the same normal distribution with mean 15.2 s and standard deviation 1.6 s. Eight of the runners have a practice race. Heidi runs the time of 13.8 s. What is the probability that she wins the race?

14 The amount of paracetamol in a tablet is distributed normally, with mean 500 mg and variance $6400\,mg^2$. The minimum dose required for relieving a headache is 380 g. Find the probability that, in a trial of 25 randomly selected participants, more than two receive less than the required dose.

15 A dice is biased and the probability distribution of the score is given in the following table.

Score	1	2	3	4	5	6
Probability	0.15	0.25	0.08	0.17	0.15	0.20

a Find the expected score for this dice.

b The dice is rolled twice. Find the probability of getting one 5 and one 6.

c The dice is rolled 10 times. Find the probability of getting at least two 1s.

16 A biased four-sided dice is used in a game. The probabilities of different outomes on the dice are given in this table.

Value	1	2	3	4
Probability	$\frac{1}{3}$	$\frac{1}{4}$	a	b

A player pays three counters to play the game, rolls the dice once and receives the number of counters equal to the number shown on the dice. Given that the player expects to lose one counter when he plays the game two times, find the value of a.

17 The heights of pupils at a school can be modelled by a normal distribution with mean 148 cm and standard deviation 8 cm.

a Find the interquartile range of the distribution.

b What percentage of the pupils at the school should be considered outliers?

18 The mean time taken to complete a test is 10 minutes and the standard deviation is 5 minutes. Explain why a normal distribution would be an inappropriate model for the time taken.

19 Quality control requires that no more than 2.5% of bottles of water contain less than the labelled volume. If a manufacturer produces bottles containing a volume of water following a normal distribution with mean value of 330 ml and standard deviation 5 ml, what should the labelled volume be, given to the nearest whole number of millilitres?

20 Jan plays a game where she tosses two fair six-sided dice. She wins a prize if the sum of her scores is 5.

a Jan tosses the two dice once. Find the probability that she wins a prize.

b Jan tosses the two dice eight times. Find the probability that she wins three prizes.

21 The time taken for a student to complete a task is normally distributed with a mean of 20 minutes and a standard deviation of 1.25 minutes.

a A student is selected at random. Find the probability that the student completes the task in less than 21.8 minutes.

b The probability that a student takes between k and 21.8 minutes is 0.3. Find the value of k.

22 The weight, W, of bags of rice follows a normal distribution with mean 1000 g and standard deviation 4 g.

a Find the probability that a bag of rice chosen at random weighs between 990 g and 1004 g.

95% of the bags of rice weigh less than k grams.

b Find the value of k.

For a bag of rice chosen at random, $P(1000 - a < W < 1000 + a) = 0.9$.

c Find the value of a.

23 A test has five questions. To pass the test, at least three of the questions must be answered correctly.

The probability that Mark answers a question correctly is $\frac{1}{5}$. Let X be the number of questions that Mark answers correctly.

a **i** Find $E(X)$.

ii Find the probability that Mark passes the test.

Bill also takes the test. Let Y be the number of questions that Bill answers correctly. The following table is the probability distribution for Y.

y	0	1	2	3	4	5
$P(Y = y)$	0.67	0.05	$a + 2b$	$a - b$	$2a + b$	0.04

b **i** Show that $4a + 2b = 0.24$.

ii Given that $E(Y) = 1$, find a and b.

c Find which student is more likely to pass the test.

24 The distances thrown by Josie in an athletics competition is modelled by a normal distribution with mean 40 m and standard deviation 5 m. Any distance less than 40 m gets 0 points. Any distance between 40 m and 46 m gets 1 point. Any distance above 46 m gets 4 points.

a Find the expected number of points Josie gets if she throws

i once

ii twice.

b What assumptions have you made in **a ii**? Comment on how realistic these assumptions are.

25 When a fair six-sided dice is rolled n times, the probability of getting no 6s is 0.194, correct to three significant figures. Find the value of n.

26 Find the smallest number of times that a fair coin must be tossed so that the probability of getting no heads is smaller than 0.001.

27 The probability of obtaining 'tails' when a biased coin is tossed is 0.57. The coin is tossed 10 times. Find the probability of obtaining

a **at least** four tails

b the fourth tail on the 10th toss.

28 A group of 100 people are asked about their birthdays. Find the expected number of dates on which no people have a birthday. You may assume that there are 365 days in a year and that people's birthdays are equally likely to be on any given date.

29 A private dining chef sends out invitations to an exclusive dinner club. From experience he knows that only 50% of those invited turn up. He can only accommodate four guests. On the first four guests he makes \$50 profit per guest; however, if more than four guests turn up he has to turn the additional guests away, giving them a voucher allowing them to have their next dinner for free, costing him \$100 per voucher.

a Assuming that responses are independent, show that his expected profit if he invites five people is \$120 to 3 significant figures.

b How many invitations should he send out?

Answer:

1 a $k = 0.5$ b 0.6 c 2.9

2 a 0.296 b 0.323

3 a 0.347 b 0.227

4 215

5 0.346

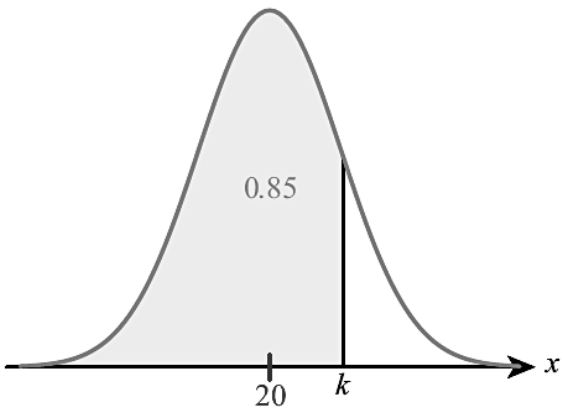
6 a $\frac{1}{2}, \frac{1}{3}, \frac{1}{6}$

b no; expected outcome is not 0

7 $N = 7$

8 a 0.933

b i



ii $k = 23.1$

9 a $P = \frac{1}{3}$ b 0.153 c 0.0435

10 a 0.206 b 0.360

11 a 0.356

b a box of six containing exactly one VL egg

12 a 0.354 b 0.740

13 0.227

14 0.232

15 a 3.52 b 0.06 c 0.456

16 $a = \frac{1}{6}$

17 a 10.8 cm b 0.698%

18 22.8% of times would be negative.

19 320 ml

20 a $\frac{1}{9}$ b 0.0426

21 a 0.925 b $k = 20.4$

22 a 0.835 b $k = 1006.58$ c $a = 6.58$

23 a i 1 ii 0.0579

b ii $a = 0.05, b = 0.02$

c Bill (0.19)

24 a i 0.845 ii 1.69

b That Josie's second throw has the same distribution as the first and is independent. These seem unlikely.

25 $n = 9$

26 10

27 a 0.919 b 0.0561

28 277

29 b 6

1 A curve has equation $y = 4x^2 - x$.

a Find $\frac{dy}{dx}$.

b Find the coordinates of the point on the curve where the gradient equals 15.

2 **a** Use technology to sketch the curve $y = f(x)$ where $f(x) = x^2 - x$.

b State the interval in which the curve is increasing.

c Sketch the graph of $y = f'(x)$.

3 The function f is given by $f(x) = 2x^3 + 5x^2 + 4x + 3$.

a Find $f'(x)$.

b Calculate the value of $f'(-1)$.

c Find the equation of the tangent to the curve $y = f(x)$ at the point $(-1, 2)$.

4 Use technology to suggest the limit of $\frac{\ln(1+x) - x}{x^2}$ as x gets very small.

5 Find the equation of the tangent to the curve $y = x^3 - 4$ at the point where $y = 23$.

6 Water is being poured into a water tank. The volume of the water in the tank, measured in m^3, at time t minutes is given by $V = 50 + 12t + 5t^2$.

a Find $\frac{dV}{dt}$.

b Find the values of V and $\frac{dV}{dt}$ after 6 minutes. What do these values represent?

c Is the volume of water in the tank increasing faster after 6 minutes or after 10 minutes?

7 The accuracy of an x-ray (A) depends on the exposure time (t) according to $A = t(2 - t)$ for $0 < t < 2$.

a Find an expression for the rate of change of accuracy with respect to time.

b At $t = 0.5$. find:

i the accuracy of the x-ray

ii the rate at which the accuracy is increasing with respect to time.

c Find the interval in which A is an increasing function.

8 Consider the function $f(x) = 0.5x^2 - \frac{8}{x}$, $x \neq 0$.

a Find $f(-2)$.

b Find $f'(x)$.

c Find the gradient of the graph of f at $x = -2$.

Let T be the tangent to the graph of f at $x = -2$.

d Write down the equation of T.

e Sketch the graph of f for $-5 \leqslant x \leqslant 5$ and $-20 \leqslant y \leqslant 20$.

f Draw T on your sketch.

The tangent, T, intersects the graph of f at a second point, P.

g Use your graphic display calculator to find the coordinates of P.

9 A curve has equation $y = 2x^3 - 8x + 3$. Find the coordinates of two points on the curve where the gradient equals -2.

10 A curve has equation $y = x^3 - 6x^2$.

a Find the coordinates of the two points on the curve where the gradient is zero.

b Find the equation of the straight line which passes through these two points.

11 A curve has equation $y = 3x^2 + 6x$.

a Find the equations of the tangents at the two points where $y = 0$.

b Find the coordinates of the point where those two tangents intersect.

12 The gradient of the curve $y = 2x^2 + c$ at the point $(p, 5)$ equals -8. Find the values of p and c.

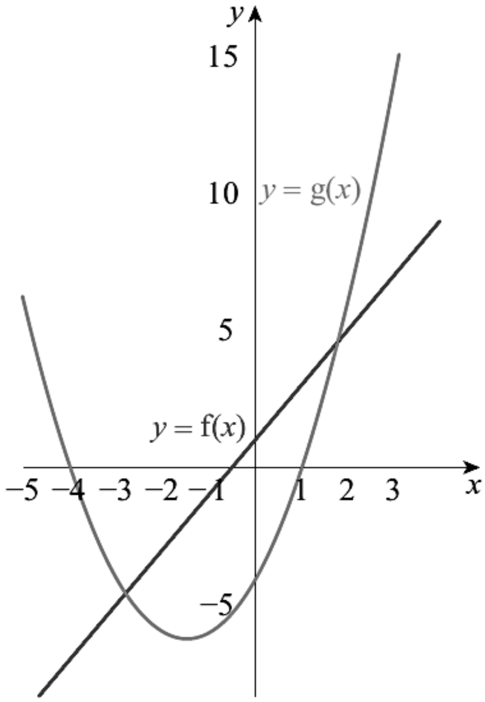
13 A company's monthly profit, \$$P$, varies according to the equation $P(t) = 15t^2 - t^3$, where t is the number of months since the foundation of the company.

a Find $\frac{dP}{dt}$.

b Find the value of $\frac{dP}{dt}$ when $t = 6$ and when $t = 12$.

c Interpret the values found in part **b**.

14 The diagram shows the graphs of the functions $f(x) = 2x + 1$ and $g(x) = x^2 + 3x - 4$.



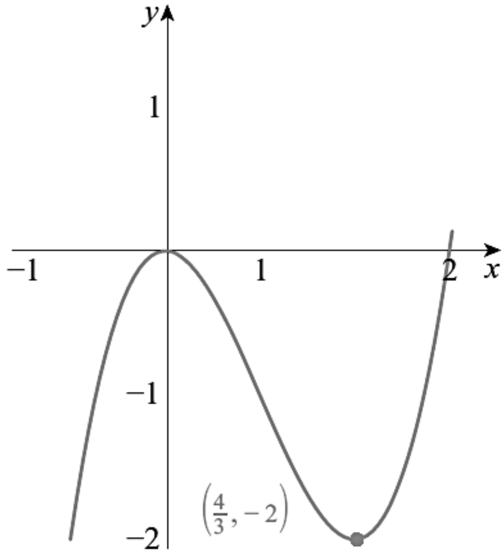
a **i** Find $f'(x)$.

ii Find $g'(x)$.

b Calculate the value of x for which the gradient of the two graphs is the same.

c On a copy of the diagram above, sketch the tangent to $y = g(x)$ for this value of x, clearly showing the property in part **b**.

15 Consider the graph on the right:



a If the graph represents $y = f(x)$:

i use inequalities to describe the interval in which $f(x)$ is decreasing

ii sketch $y = f'(x)$.

b If the graph represents $y = f'(x)$:

i use inequalities to describe the interval in which $f(x)$ is decreasing

ii sketch a possible graph of $y = f(x)$.

16 The line l passes through the points $(-1, -2)$ and $(1, 4)$. The function f is given by $f(x) = x^2 - x + 2$.

a Find the gradient of l.

b Differentiate $f(x)$ with respect to x.

c Find the coordinates of the point where the tangent to $y = f(x)$ is parallel to the line l.

d Find the coordinates of the point where the tangent to $y = f(x)$ is perpendicular to the line l.

e Find the equation of the tangent to $y = f(x)$ at the point $(3, 8)$.

f Find the coordinates of the vertex of $y = f(x)$ and state the gradient of the curve at this point.

17 Given that $f(x) = x^2 + x - 5$:

a write down $f'(x)$

b find the values of x for which $f'(x) = f(x)$.

18 The gradient of the curve $y = ax^2 + bx$ at the point $(2, -2)$ is 3. Find the values of a and b.

19 The gradient of the normal to the curve with equation $y = ax^2 + bx$ at the point $(1, 5)$ is $\frac{1}{3}$. Find the values of a and b.

20 For the curve with equation $y = 5x^2 - 4$, find the coordinates of the point where the tangent at $x = 1$ intersects the tangent at the point $x = 2$.

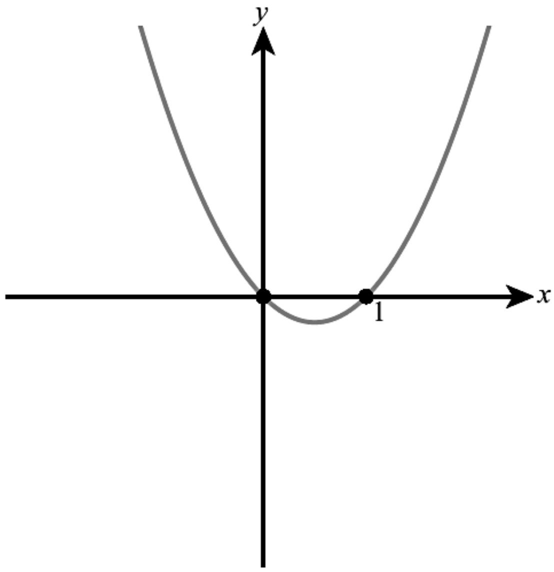
21 Let $f(x) = \frac{\ln(4x)}{x}$, for $0 < x \leqslant 5$.

Points $P(0.25, 0)$ and Q are on the curve of f. The tangent to the curve of f at P is perpendicular to the tangent at Q. Find the coordinates of Q.

Answer:

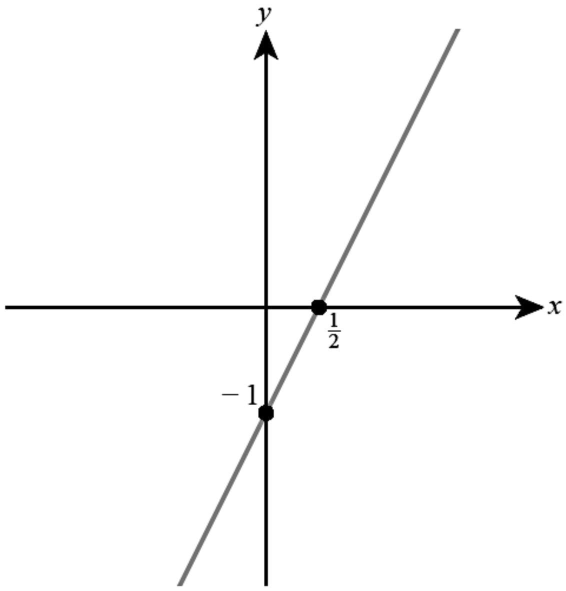
1 a $8x-1$ b $(2, 15)$

2 a



b $x > \frac{1}{2}$

c



3 a $f'(x) = 6x^2 + 10x + 4$

b $f'(-1) = 0$ c $y = 2$

4 -0.5

5 $y = 27x - 58$

6 a $12 + 10t$

b $V(6) = 302$, $\frac{dV}{dt}(6) = 72$

After 6 minutes, there is $302\,m^3$ of water in the tank, and the volume of water in the tank is increasing at a rate of $72\,m^3$ per minute.

c $\frac{dV}{dt}(10) = 112$, so the volume is increasing faster after 10 minutes than after 6 minutes.

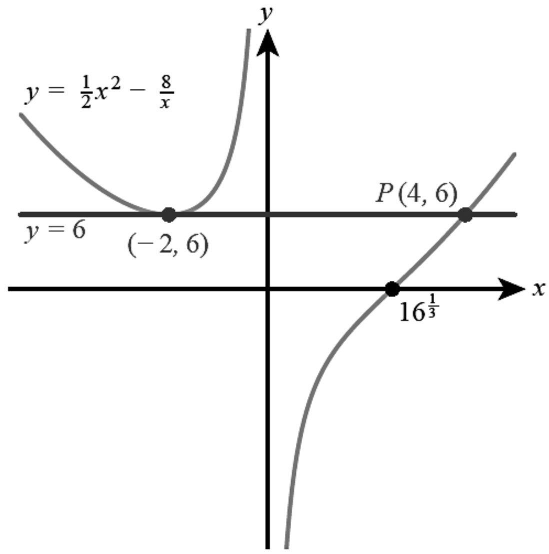
7 a $2-2t$ b i 0.75 ii 1

c $0 < t < 1$

8 a 6 b $x + 8x^{-2}$

c 0 d $y = 6$

e, f



g $(4, 6)$

9 $(1, -3)$, $(-1, 9)$

10 a $(0, 0)$, $(4, -32)$ b $y = -8x$

11 a $y = 6x$, $y = -6x - 12$

b $(-1, -6)$

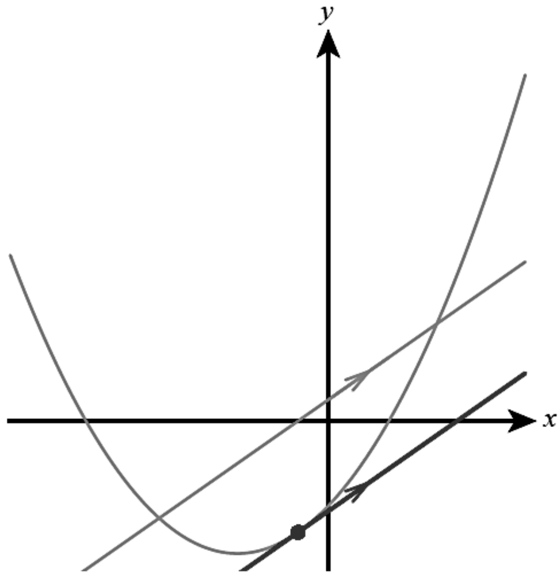
12 $p = -2$, $c = -3$

13 a $30t - 3t^2$ b $72, -72$

c The profit is increasing after 6 months, but decreasing after 12 months.

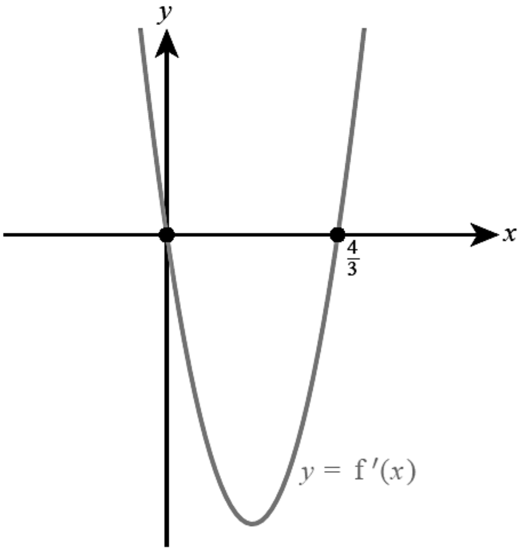
14 a i 2 ii $2x + 3$ b $-\frac{1}{2}$

c



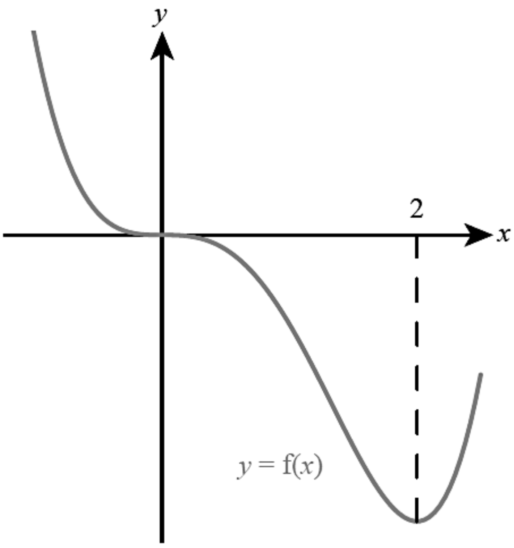
15 a i $0 < x < \frac{4}{3}$

ii



b i $x < 0$ and $0 < x < 2$

ii



16 a 3 b $2x - 1$

c $(2, 4)$ d $\left(\frac{1}{3}, \frac{16}{9}\right)$

e $y = 5x - 7$ f $\left(\frac{1}{2}, \frac{7}{4}\right)$, gradient $= 0$

17 a $2x + 1$ b 3 and -2

18 $a = 2, b = -5$

19 $a = -8, b = 13$

20 $(1.5, 6)$

21 $(0.701, 1.47)$

You need to make good use of technology in this question!

1 Find $\int x^3 - \frac{3}{x^2}\,dx$.

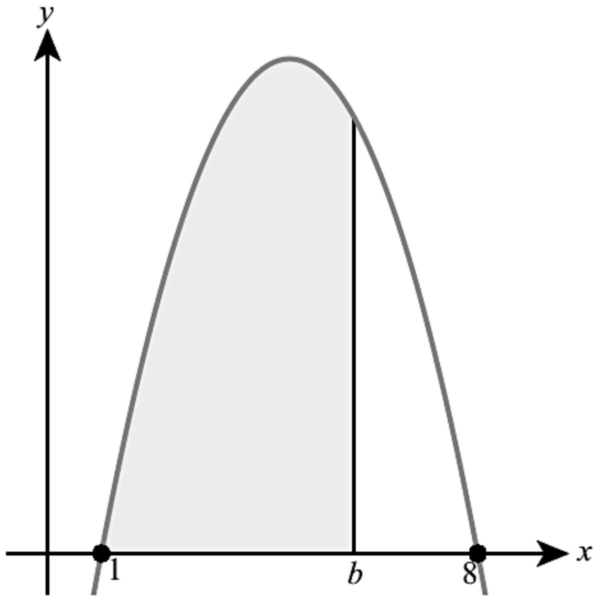
2 Find $\int 4x^2 - 3x + 5\,dx$.

3 Evaluate $\int_1^5 \frac{2}{x^4}\,dx$.

4 A curve has gradient given by $\frac{dy}{dx} = 3x^2 - 8x$ and passes through the point (1, 3). Find the equation of the curve.

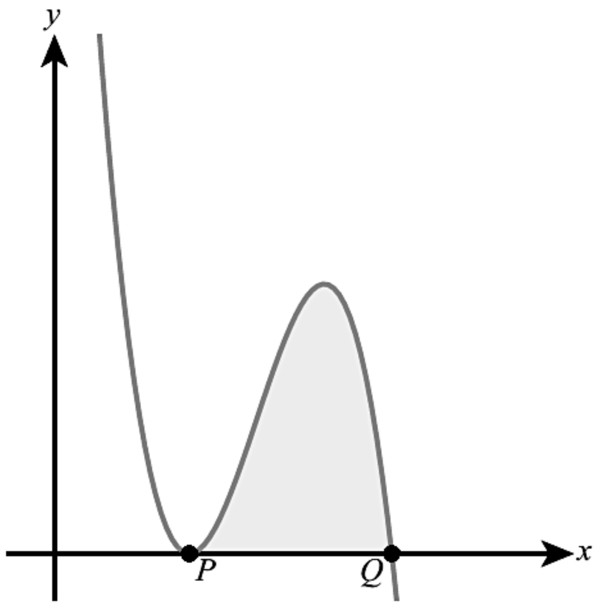
5 Find the value of a such that $\int_2^a 2 - \frac{8}{x^2}\,dx = 9$.

6 The graph of $y = 9x - x^2 - 8$ is shown in the diagram.



Given that the shaded area equals 42.7, find the value of b correct to one decimal place.

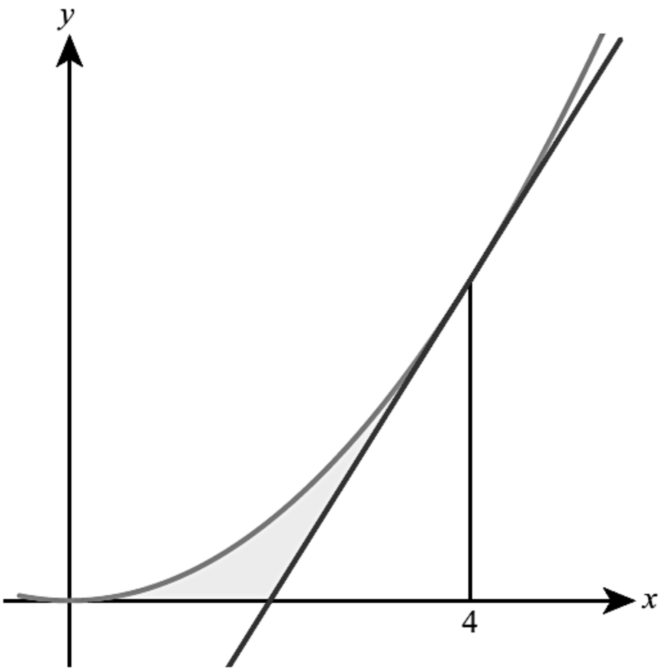
7 The diagram shows the graph of $y = -x^3 + 9x^2 - 24x + 20$.



a Find the coordinates of P and Q.

b Find the shaded area.

8 The diagram shows the curve with equation $y = 0.2x^2$ and the tangent to the curve at the point where $x = 4$.

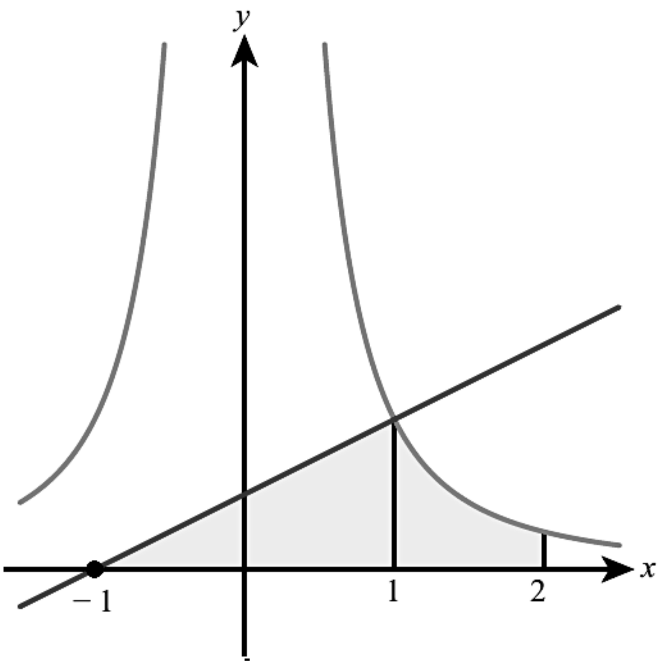


a Find the equation of the tangent.

b Show that the tangent crosses the x-axis at the point (2, 0).

c Find the shaded area.

9 The diagram shows the graph of $y = \frac{1}{x^2}$ and the normal at the point where $x = 1$.



a Show that the normal crosses the x-axis at $(-1, 0)$.

b Find the shaded area.

10 The gradient of the normal to the curve $y = f(x)$ at any point equals x^2. If $f(1) = 2$, find $f(2)$.

11 The rate at which water is accumulated in a rainwater measuring device in a storm is given by $20t\,\text{cm}^3/\text{minute}$.

If the container is initially empty, find the volume of water in the container after 10 minutes.

12 A puppy at age 6 months has a mass of 2.3 kg. Its growth after this point is modelled by $\frac{A}{20} + c$ kg/month where A is the puppy's age in months.

When the puppy is 10 months old it is growing at a rate of 1.5 kg per month. Use this model to estimate the mass of the puppy when it is 18 months old.

13 The function $f(x)$ is such that $f'(x) = 3x^2 + k$ where k is a constant. If $f(1) = 13$ and $f(2) = 24$, find $f(3)$.

14 A nutritionist designs an experiment to find the energy contained in a nut. They burn the nut and model the energy emitted as $\frac{k}{t^2}$ calories for $t > 1$ where t is in seconds.

They place a beaker of water above the burning nut and measure the energy absorbed by the water using a thermometer.

a State one assumption that is being made in using this experimental setup to measure the energy content of the nut.

b When $t = 1$ the water has absorbed 10 calories of energy. When $t = 2$ the water has absorbed 85 calories of energy. Use technology to estimate the total energy emitted by the nut if it is allowed to burn indefinitely.

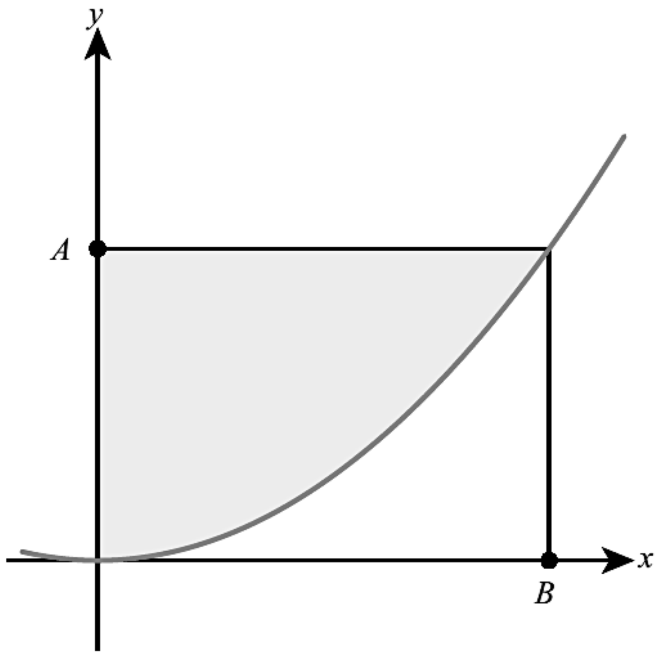
15 The gradient at every point on a curve is proportional to the square of the x-coordinate at that point.

Find the equation of the curve, given that it passes through the points (0, 3) and $\left(1, \frac{14}{3}\right)$.

16 The function f satisfies $f'(x) = 3x - x^2$ and $f(4) = 0$. Find $\int_0^4 f(x)\,dx$.

17 Given that $\int f(x)\,dx = 3x^2 - \frac{2}{x} + c$, find an expression for $f(x)$.

18 The diagram shows a part of the curve with equation $y = x^2$. Point A has coordinates (0, 9).



a Find the coordinates of point B.

b Find the shaded area.

Answer:

1 $\frac{x^4}{4} + \frac{3}{x} + c$

2 $\frac{4}{3}x^3 - \frac{3}{2}x^2 + 5x + c$

3 0.661

4 $y = x^3 - 4x^3 + 6$

5 $a = 8$

6 5.7

7 a (2, 0), (5, 0) b 6.75

8 a $y = 1.6x - 3.2$ c 1.07

9 b 1.5

10 1.5

11 1000 cm^3

12 21.5 kg

13 47

14 a e.g. no energy lost to surroundings.
b 160 calories

15 $y = \frac{5x^3}{3} + 3$

16 0

17 $6x + \frac{2}{x^2}$

18 a (3, 0) b 18

1 A librarian is investigating the number of books borrowed from the school library over a period of 10 weeks. She decided to select a sample of 10 days and record the number of books borrowed on that day.

a She first suggests selecting a day at random and then selecting every seventh day after that.

i State the name of this sampling technique.

ii Identify one possible source of bias in this sample.

b The librarian changes her mind and selects a simple random sample of 10 days instead.

i Explain what is meant by a simple random sample in this context.

ii State one advantage of a simple random sample compared to the sampling method from part **a**.

c For the days in the sample, the numbers of books borrowed were:

17, 16, 21, 16, 19, 20, 18, 11, 22, 14

Find

i the range of the data

ii the mean number of books borrowed per day

iii the standard deviation of the data.

2 The table shows the maximum temperature (T°C) and the number of cold drinks (n) sold by a small shop on a random sample of nine summer days.

T	21	28	19	21	32	22	27	18	30
n	20	37	21	18	35	25	31	17	38

a Using technology, or otherwise, plot the data on a scatter graph.

b Describe the relationship between the temperature and the sales of cold drinks.

c Find the equation of the regression line of n on T.

d Use your regression line to estimate the number of cold drinks sold on the day when the maximum temperature is 26° C.

3 The masses of 50 cats are summarized in the grouped frequency table:

Mass (kg)	$1.2 \leqslant m < 1.6$	$1.6 \leqslant m < 2.0$	$2.0 \leqslant m < 2.4$	$2.4 \leqslant m < 2.8$	$2.8 \leqslant m < 3.2$
Frequency	4	10	8	16	12

a Use this table to estimate the mean mass of a cat in this sample. Explain why your answer is only an estimate.

b Use technology to create a cumulative frequency graph.

c Use your graph to find the median and the interquartile range of the masses.

d Create a box plot to represent the data. You may assume that there are no outliers.

4 A survey was carried out on a road to determine the number of passengers in each car (excluding the driver). The table shows the results of the survey.

Number of passengers	0	1	2	3	4
Number of cars	37	23	36	15	9

a State whether the data are discrete or continuous.

b Write down the mode.

c Use your GDC to find

i the mean number of passengers per car

ii the median number of passengers per car

iii the standard deviation.

5 Two groups of 40 students were asked how many books they have read in the last two months. The results for **the first group** are shown in the following table.

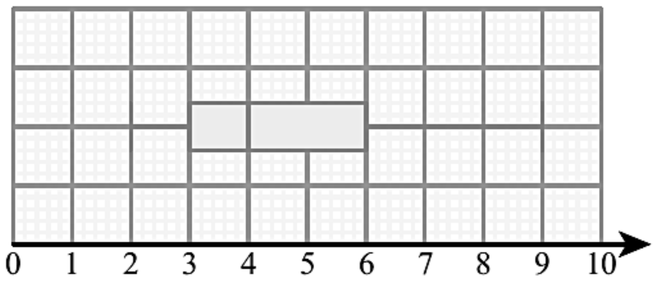
Number of books read	Frequency
2	5
3	8
4	13
5	7
6	4
7	2
8	1

The quartiles for these results are 3 and 5.

a Write down the value of the median for these results.

b Draw a box-and-whisker diagram for these results.

The results for **the second group** of 40 students are shown in the following box-and-whisker diagram.



c Estimate the number of students **in the second group** who have read at least 6 books.

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6 The following table shows the Diploma score x and university entrance mark y for seven IB Diploma students.

Diploma score (x)	28	30	27	31	32	25	27
University entrance mark (y)	73.9	78.1	70.2	82.2	85.5	62.7	69.4

a Find the correlation coefficient.

The relationship can be modelled by the regression line with equation $y = ax + b$.

b Write down the value of a and b.

Rita scored a total of 26 in her IB Diploma.

c Use your regression line to estimate Rita's university entrance mark.

7 A student recorded, over a period of several months, the amount of time he waited in the queue for lunch at the college canteen. He summarized the results in this cumulative frequency table.

Time (minutes)	$\leqslant 2$	$\leqslant 4$	$\leqslant 6$	$\leqslant 8$	$\leqslant 10$	$\leqslant 12$
Cumulative frequency	4	9	16	37	45	48

a Draw a cumulative frequency graph for the data.

b Use your graph to estimate

i the median

ii the interquartile range of the times

iii the 90th percentile.

c Complete the grouped frequency table:

Time (min)	$0 < t \leqslant 2$	$0 < t \leqslant 4$				
Frequency						

d Estimate the mean waiting time.

8 The number of customers visiting a shop is recorded over a period of 12 days:
26, 33, 28, 47, 52, 45, 93, 61, 37, 55, 57, 34

a Find the median and the quartiles.
b Determine whether there are any outliers.
c Draw a box-and-whisker diagram for the data.

9 The heights of a group of 7 children were recorded to the nearest centimetre:
127, 119, 112, 123, 122, 126, 118

a Find the mean and the variance of the heights.
b Each child stands on a 35-centimetre-high stool. Find the mean and variance of their heights.

10 Theo is keeping a record of his travel expenses. The cost of each journey is \$15 plus \$3.45 per kilometre. The mean length of Theo's journeys is 11.6 km and the standard deviation of the lengths is 12.5 km. Find the mean and standard deviation of his cost per journey.

11 The frequency table summarizes data from a sample with mean 1.6. Find the value of x.

x	0	1	2	3
Frequency	5	6	8	x

12 A scientist measured a sample of 12 adult crabs found on a beach, measuring their shell length (s) and mass (m).

Shell length (cm)	Mass (g)
7.1	165
8.1	256
8.5	194
6.0	150
9.0	275
5.3	204

Shell length (cm)	Mass (g)
5.9	143
8.4	190
9.2	208
5.1	194
6.3	217
9.1	268

a Using technology or otherwise plot a scatter diagram to illustrate the scientist's results.
b The scientist later realized that the beach contains two species of crab – the Lesser European Crab and the Giant European Crab. Her research suggests that the Giant European crab tends to be heavier than similar-sized Lesser European Crabs. Find the equation for a regression line for the mass of the Giant European crab if its shell length is known.
c Find the correlation coefficient for the data for the Giant European crab and comment on your result.
d Estimate the mass of a Giant European Crab with shell length 8 cm.
e Juvenile Giant European Crabs have a shell length of between 2 and 4 cm. Estimate the possible masses of these crabs and comment on the reliability of your results.

13 The one hour distances, in miles, covered by runners before (x) and after (y) going on a new training program are recorded.

The correlation between these two distances is found to be 0.84. The regression line is $y = 1.2x + 2$.

a Describe the significance of
  i the intercept of the regression line being positive
  ii the gradient of the regression line being greater than 1.
b If the previous mean distance is 8 miles, find the new mean.

Their trainer wants to have the data in km. To convert miles to km all the distances are multiplied by 1.6. The new variables in km are X and Y.

c Find
  i the correlation between X and Y
  ii the regression line connecting X and Y.

Answer:

1 a i systematic sampling

ii e.g. Students may take books out on the same day each week.

b i Each possible sample of 10 days has an equal chance of being selected.

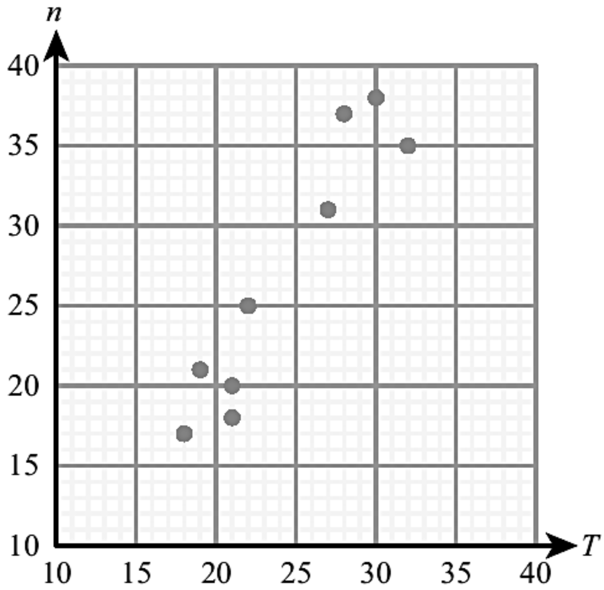
ii Representative of the population of all days.

c i 11

ii 17.4

iii 3.17

2 a



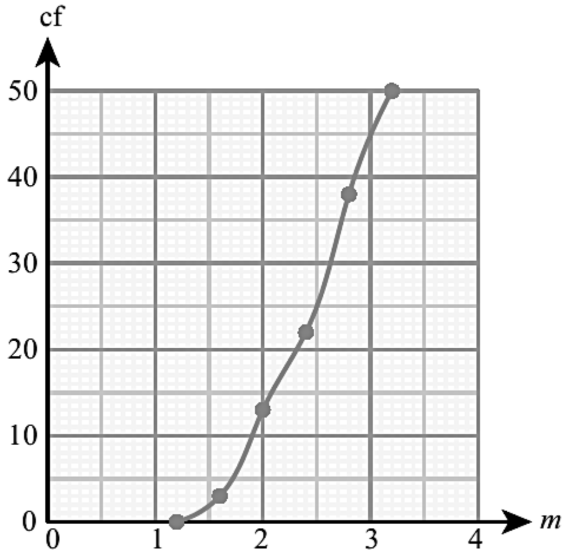
b strong positive correlation

c $n = 1.56T - 10.9$

d 30.0

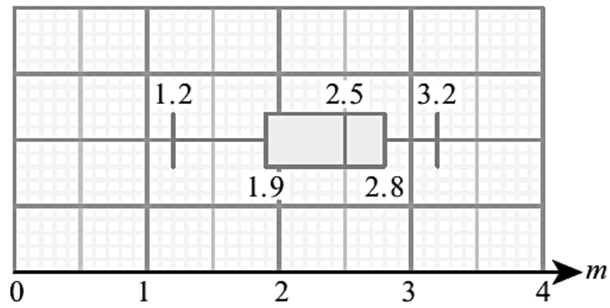
3 a 2.38 kg

b



c 2.5 kg, 0.9 kg

d



4 a discrete

b 0

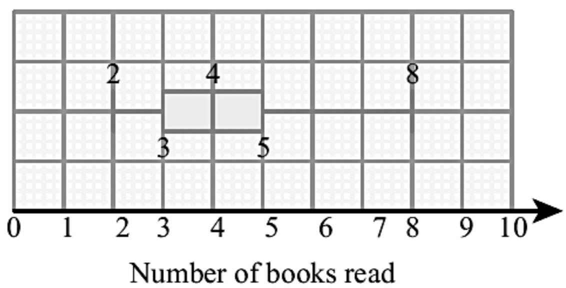
c i 1.47

ii 1.5

iii 1.25

5 a 4

b



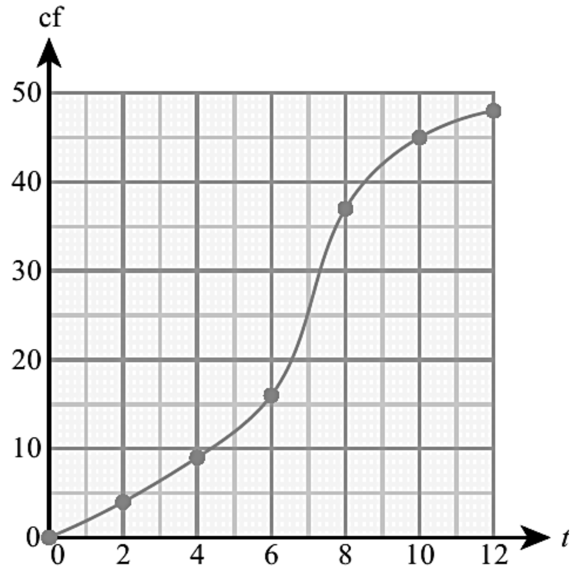
c 10

6 a 0.996

b $a = 3.15$, $b = -15.4$

c 66.5

7 a



b i 6.9 minutes

ii 2.9 minutes

iii 9.3 minutes

c

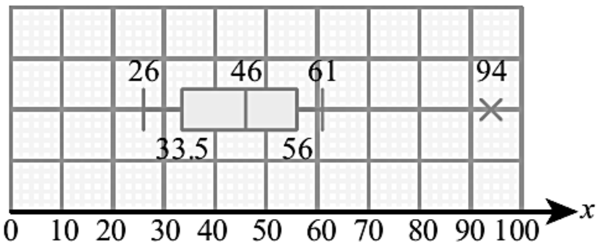
Time	Freq
$0 \leqslant t < 2$	4
$2 \leqslant t < 4$	5
$4 \leqslant t < 6$	7
$6 \leqslant t < 8$	11
$8 \leqslant t < 10$	8
$10 \leqslant t < 12$	3

d 6.21 minutes

8 a med = 46, $Q_1 = 33.5$, $Q_3 = 56$

b yes (93)

c



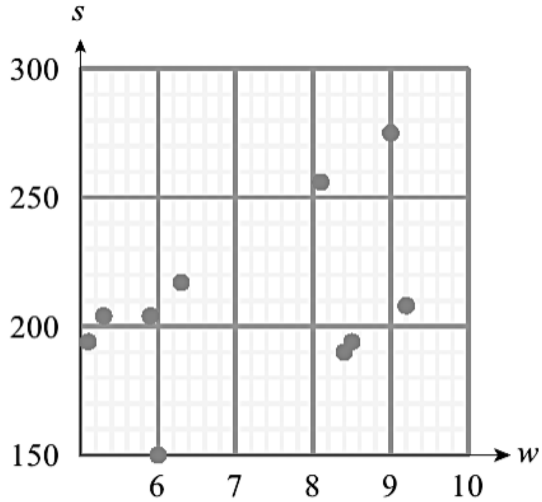
9 a 121 cm, 22.9 cm^2

b 156 cm, 22.9 cm^2

10 mean = \$55.02, sd = \$43.13

11 $x = 6$

12 a



b $w = 19.1s + 99.0$

c 0.994 – strong positive correlation

d 252 g

e 137 g to 175 g; extrapolating from the data sc not reliable.

13 a i Athletes generally do better after the programme.

ii Better athletes improve more.

b 11.6 miles

c i 0.84

ii $Y = 1.2X + 3.2$

1 An arithmetic sequence has first term 7 and third term 15.

a Find the common difference of the series.

b Find the 20th term.

c Find the sum of the first 20 terms.

2 Line L has equation $2x - 3y = 12$.

a Find the gradient of L.

b The point $P(9, k)$ lies on L. Find the value of k.

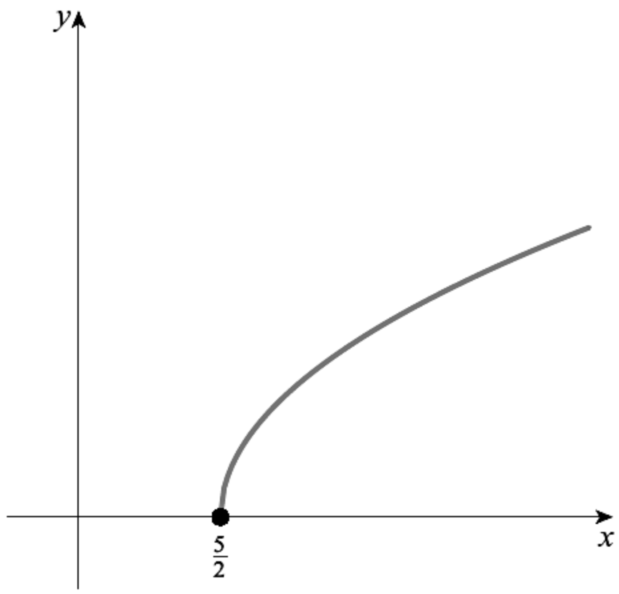
c Line M is perpendicular to L and passes through the point P. Find the equation of M in the form $ax + by = c$.

3 a Sketch the graph of $y = (x + 2)e^{-x}$, labelling all the axis intercepts and the coordinates of the maximum point.

b State the equation of the horizontal asymptote.

c Write down the range of the function $f(x) = (x + 2)e^{-x}$, $x \in \mathbb{R}$.

4 A function is defined by $f(x) = \sqrt{x - a}$. The graph of $y = f(x)$ is shown in the diagram.



a Write down the value of a and state the domain of f.

b Write down the range of f.

c Find $f^{-1}(4)$.

5 In triangle ABC, $AB = 12$ cm, $BC = 16$ cm and $AC = 19$ cm.

a Find the size of the angle ACB.

b Calculate the area of the triangle.

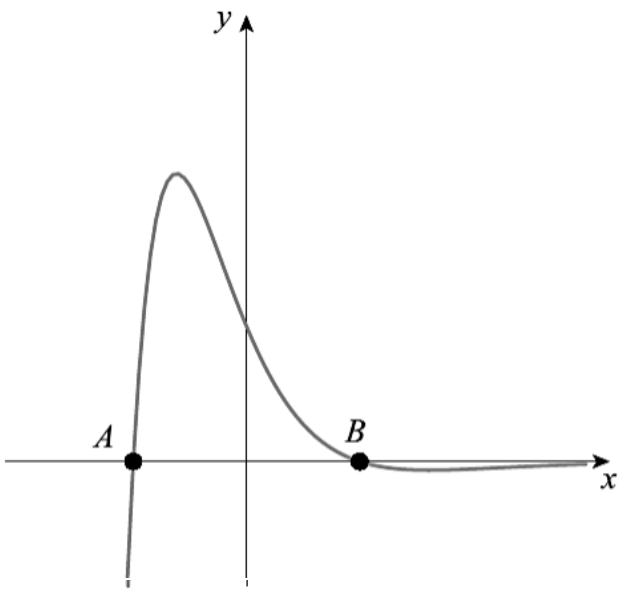
6 A random variable X follows a normal distribution with mean 26 and variance 20.25.

a Find the standard deviation of X.

b Find $P(21.0 < x < 25.3)$.

c Find the value of a such that $P(X > a) = 0.315$.

7 The curve in the diagram has equation $y = (4 - x^2)e^{-x}$.



a Find the coordinates of the points A and B.

b Find the shaded area.

8 The IB grades attained by a group of students are listed as follows.

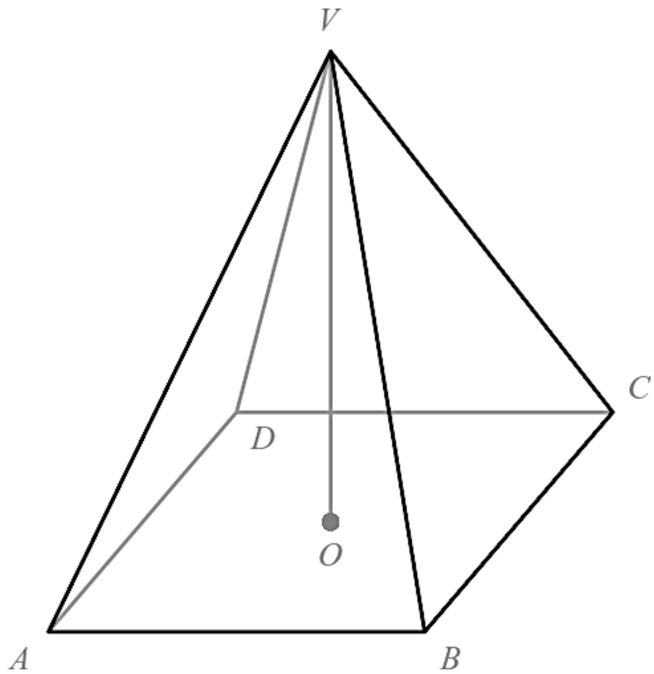
6 4 5 3 7 3 5 4 2 5

a Find the median grade.

b Calculate the interquartile range.

c Find the probability that a student chosen at random from the group scored at least a grade 4.

9 $ABCDV$ is a solid glass pyramid. The base of the pyramid is a square of side 3.2 cm. The vertical height is 2.8 cm. The vertex V is directly above the centre O of the base.



a Calculate the volume of the pyramid.

b The glass weighs 9.3 grams per cm^3. Calculate the weight of the pyramid.

c Show that the length of the sloping edge VC of the pyramid is 3.6 cm.

d Calculate the angle at the vertex $B\hat{V}C$.

e Calculate the total surface area of the pyramid.

10 a Sketch the graph of the function $f(x) = \frac{2x+3}{x+4}$, for $-10 \leqslant x \leqslant 10$, indicating clearly the axis intercepts and any asymptotes.

b Write down the equation of the vertical asymptote.

c On the same diagram sketch the graph of $g(x) = x + 0.5$.

d Using your graphical display calculator write down the coordinates of one of the points of intersection on the graphs of f and g, giving your answer correct to five decimal places.

e Write down the gradient of the line $g(x) = x + 0.5$.

f The line L passes through the point with coordinates $(-2, -3)$ and is perpendicular to the line $g(x)$. Find the equation of L.

11 Find the equation of the normal to the curve $y = x + 3x^2$ at the point where $x = -2$. Give your answer in the form $ax + by + c = 0$.

12 a Find the value of x given that $\log_{10} x = 3$.

b Find $\log_{10} 0.01$.

13 a Expand and simplify $(2x)^3(x - 3x^{-5})$.

b Differentiate $y = (2x)^3(x - 3x^{-5})$.

14 A geometric series has first term 18 and common ratio r.

a Find an expression for the 4th term of the series.

b Write down the formula for the sum of the first 15 terms of this series.

c Given that the sum of the first 15 terms of the series is 26.28 find the value of r.

15 Theo is looking to invest £200 for 18 months. He needs his investment to be worth £211 by the end of the 18 months. What interest rate does he need? Give your answer as a percentage correct to one decimal place.

16 The distribution of a discrete random variable X is given by $P(X = x) = \frac{k}{x^2}$ for $X = 1, 2, 3, 4$. Find $E(X)$.

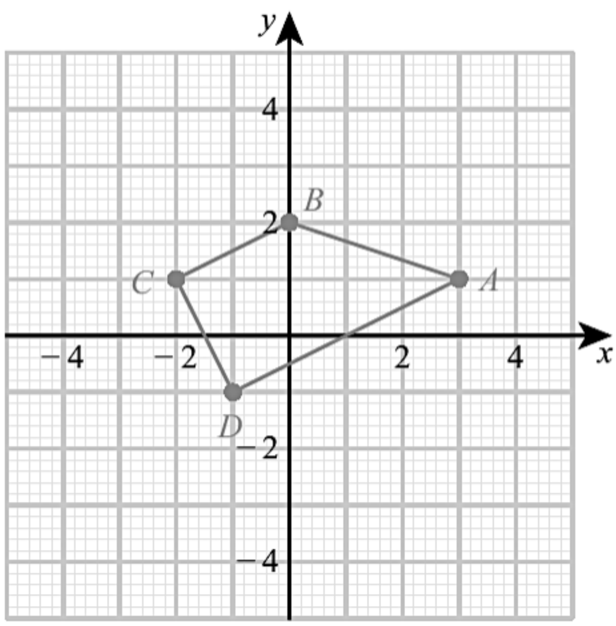
17 A box holds 240 eggs. The probability that an egg is brown is 0.05.

a Find the expected number of brown eggs in the box.

b Find the probability that there are 15 brown eggs in the box.

c Find the probability that there are at least 10 brown eggs in the box.

18 The vertices of quadrilateral $ABCD$ as shown in the diagram are $A(3, 1)$, $B(0, 2)$, $C(-2, 1)$ and $D(-1, -1)$.



a Calculate the gradient of line *CD*.

b Show that line *AD* is perpendicular to line *CD*.

c Find the equation of line *CD*. Give your answer in the form $ax + by = c$ where $a, b, c \in \mathbb{Z}$.

Lines *AB* and *CD* intersect at point *E*. The equation of line *AB* is $x + 3y = 6$.

d Find the coordinates of *E*.

e Find the distance between *A* and *D*.

The distance between *D* and *E* is $\sqrt{20}$.

f Find the area of triangle *ADE*.

19 a Jenny has a circular cylinder with a lid. The cylinder has height 39 **cm** and diameter 65 **mm**.

i Calculate the volume of the cylinder **in cm³**. Give your answer correct to **two** decimal places.

The cylinder is used for storing tennis balls.

Each ball has a **radius** of 3.25 cm.

ii Calculate how many balls Jenny can fit in the cylinder if it is filled to the top.

iii I Jenny fills the cylinder with the number of balls found in part ii and puts the lid on. Calculate the volume of air inside the cylinder in the spaces between the tennis balls.

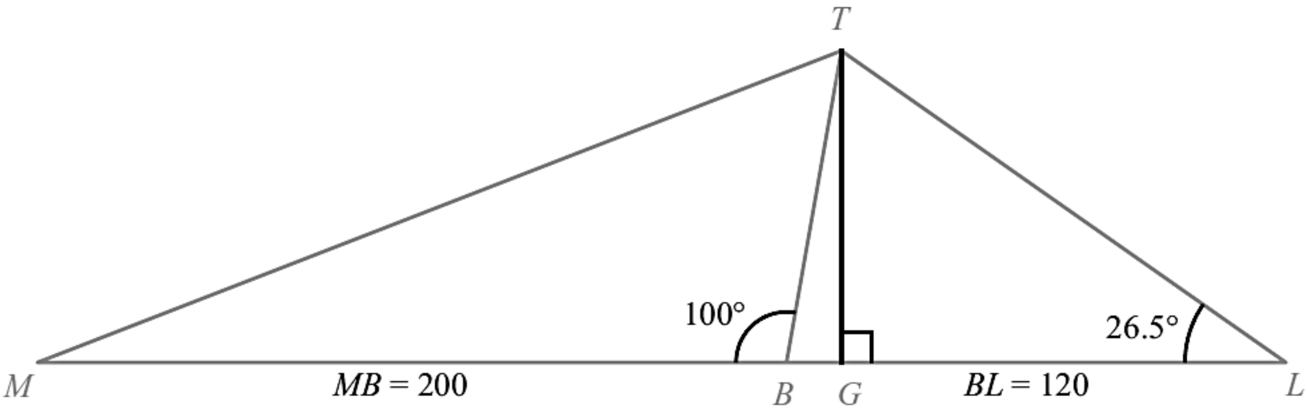
II Convert your answer to iii I into cubic metres.

b An old tower (*BT*) leans at 10° away from the vertical (represented by line *TG*).

The base of the tower is at *B* so that $M\hat{B}T = 100°$.

Leonardo stands at *L* on flat ground 120 m away from *B* in the direction of the lean.

He measures the angle between the ground and the top of the tower *T* to be $B\hat{L}T = 26.5°$.



i I Find the value of angle $B\hat{T}L$.

II Use triangle *BTL* to calculate the sloping distance *BT* from the base *B* to the top, *T* of the tower.

ii Calculate the vertical height *TG* of the top of the tower.

iii Leonardo now walks to point *M*, a distance 200 m from *B* on the opposite side of the tower. Calculate the distance from *M* to the top of the tower at *T*.

20 The following table shows the number of bicycles, *x*, produced daily by a factory and their total production cost, *y*, in US dollars (USD). The table shows data recorded over seven days.

	Day 1	Day 2	Day 3	Day 4	Day 5	Day 6	Day 7
Number of bicycles, x	12	15	14	17	20	18	21
Production cost, y	3900	4600	4100	5300	6000	5400	6000

a i Write down the Pearson's product–moment correlation coefficient, *r*, for these data.

ii Hence comment on the result.

b Write down the equation of the regression line y on x for these data, in the form $y = ax + b$.

c Estimate the total cost, to the nearest USD, of producing 13 bicycles on a particular day.

All the bicycles that are produced are sold. The bicycles are sold for 304 USD each.

d Explain why the factory does not make a profit when producing 13 bicycles on a particular day.

e i Write down an expression for the total selling price of x bicycles.

ii Write down an expression for the profit the factory makes when producing x bicycles on a particular day.

iii Find the least number of bicycles that the factory should produce, on a particular day, in order to make a profit.

21 It is known that, among all college students, the time taken to complete a test paper is normally distributed with mean 52 minutes and standard deviation 7 minutes.

a Find the probability that a randomly chosen student completes the test in less than 45 minutes.

b In a group of 20 randomly chosen college students, find the probability that:

i exactly one completes the test in less than 45 minutes

ii more than three complete the test in less than 45 minutes.

22 In a class of 26 students, 15 study French, 14 study biology and 8 study history.

Of those students, 7 study both French and biology, 4 study French and history, and 3 study biology and history.

a Using a Venn diagram, or otherwise, find how many students study all three subjects.

b Find the probability that a randomly selected student studies French only.

c Given that a student studies French, what is the probability that they do not study biology?

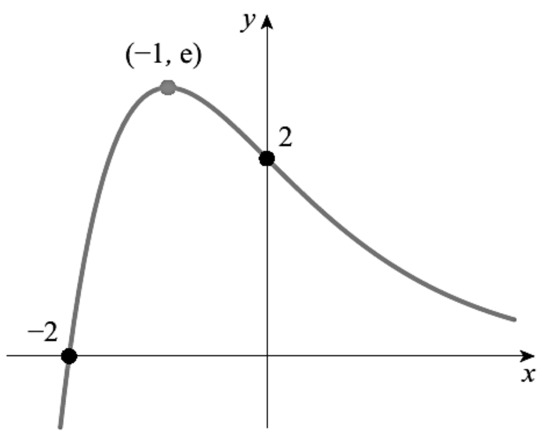
d Two students are selected at random. What is the probability that at least one of them studies history?

Answer:

1 a 4 b 83 c 900

2 a $\frac{2}{3}$ b 2 c $3x+2y=31$

3 a



b $y = 0$ c $y \leqslant 2.72$

4 a $a = 2.5; x \geqslant 2.5$ b $f(x) \geqslant 0$

c 18.5

5 a 38.9° b $95.5\,cm^2$

6 a 4.5 b 0.305

c 28.2

7 a (−2, 0), (2, 0) b 15.6

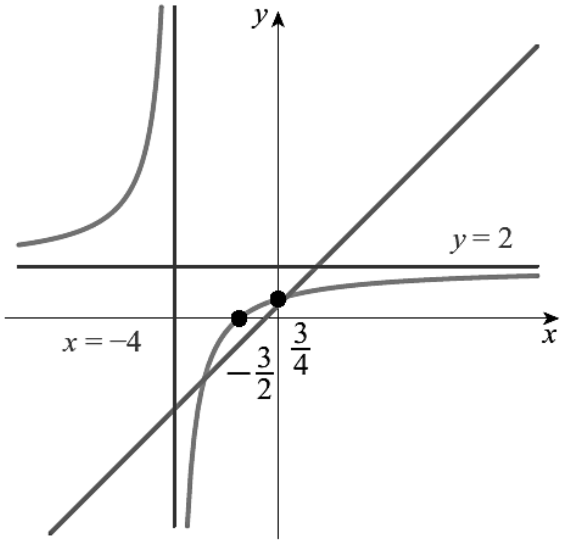
8 a 4.5 b 2

c $\frac{7}{10}$

9 a $9.56\,cm^3$ b 88.9 g

d 52.8° e $30.9\,cm^2$

10 a and c



b $x=-4$

d $(-2.85078, -2.35078)$ OR $(0.35078, 0.85078)$

e 1

f $y=-x-5$

11 $x-11y+112=0$

12 a 1000 b −2

13 a $8x^7-24x^{-2}$ b $56x^6+48x^{-3}$

14 a $18r^3$ b $\frac{18(1-r^{15})}{1-r}$

c 0.315

15 3.6%

16 $\frac{60}{41}$

17 a 12 b 0.0733

c 0.764

18 a −2

c $y=-2x-3$ d $(-3, 3)$

e $\sqrt{20}$ f 10

19 a i 1295.12 cm^3 ii 6

iii I 431 cm^3

II $4.31\times10^{-4} m^3$

b i I 73.5°

II 55.8 m

ii 55.0 m iii 217 m

20 a i 0.985

ii strong positive

b $y=260x+699$

c 4077 USD

d e.g. $3952<4077$

e i $304x$

ii $304x-(260x+699)$

iii 16

21 a 0.159

b i 0.119

ii 0.394

22 a 3

b $\frac{7}{26}$

c $\frac{8}{15}$

d $\frac{172}{325}$

1 The base of a triangle is 6 cm larger than its perpendicular height. The area is 20 cm^2. Find the height.

2 **a** Show that the equation
$(2x + 5)(x - 4) = 10$
can be written in the form
$ax^2 + bx + c = 0$
where a, b, and c are whole numbers to be found.

b Hence solve the equation
$(2x + 5)(x - 4) = 10$

3 **a** Show that the equation
$$x^2 - 7 = \frac{2}{x+4}$$
can be written in the form
$ax^3 + bx^2 + cx + d = 0$
where a, b, c and d are whole numbers to be found.

b Hence solve the equation
$$x^2 - 7 = \frac{2}{x+4}$$

4 Two numbers differ by 1. The product of the numbers is 10. The smaller number is x.

a Write this information as a quadratic equation in the form $ax^2 + bx + c = 0$

b Hence find the possible values of x to three significant figures.

5 A square-based cuboid has one side 2 cm longer than the other two. The volume of the cuboid is 10 cm^3. Find the length of the shortest side of the cuboid to three significant figures.

6 A group of insects contains only flies and spiders. Flies have six legs and spiders have eight legs, but both insects have one head. There are 142 legs and 20 heads in total.

a Write down simultaneous equations to describe this situation.

b How many spiders are there?

7 Jack and Jill buy 'pails of water' and 'vinegar and brown paper'.
Jack buys 7 pails and 5 vinegar and brown papers. He spends $70.
Jill buys 5 pails and 7 vinegar and brown papers. She spends $74.
How much more expensive is vinegar and brown paper than a pail of water?

8 Daniel and Alessia are buying books. Science books cost £8 and art books cost £12. Daniel spends £168 on books. Alessia buys twice as many art books than Daniel but half as many science books than Daniel. She spends £264. How many books do they buy in total?

9 10 000 people attended a sports match. Let x be the number of adults attending the sports match and y be the number of children attending the sports match.

a Write down an equation in x and y.

The cost of an adult ticket was 12 Australian dollars (AUD). The cost of a child ticket was 5 Australian dollars (AUD).

b Find the total cost for a family of 2 adults and 3 children.

The total cost of tickets sold for the sports match was 108 800 AUD.

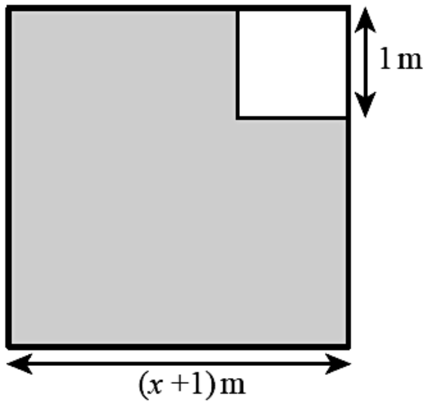
c Write down a second equation in x and y.

d Write down the value of x and the value of y.

10 In an arithmetic sequence, $S_{40} = 1900$ and $u_{40} = 106$. Find the value of u_1 and of d.

11 The length of a square garden is $(x + 1)$ m. In one of the corners a square 1 m in length is used only for grass. The rest of the garden is only for planting roses and is shaded in the diagram opposite.
The area of the shaded region is A.

a Write down an expression for A in terms of x.
b Find the value of x given that $A = 109.25\,\text{m}^2$.
c The owner of the garden puts a fence around the shaded region. Find the length of this fence.



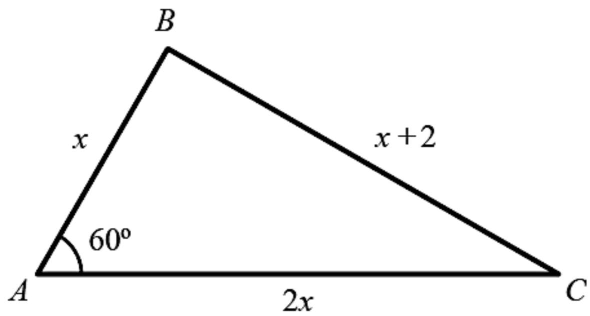
12 The product of three consecutive integers is 504. Find the mean of the numbers.

13 A geometric sequence has first term 8 and the sum of the first three terms is 14. Find the possible values of the common ratio.

14 A geometric sequence has first term 2 and the sum of the first four terms is −40. Find the common ratio.

15 The triangle ABC has $AB = x$, $BC = x + 2$, $AC = 2x$ and $B\hat{A}C = 60°$ as shown opposite.

a Show that $x^2 - 2x - 2 = 0$
b Hence find the value of x.



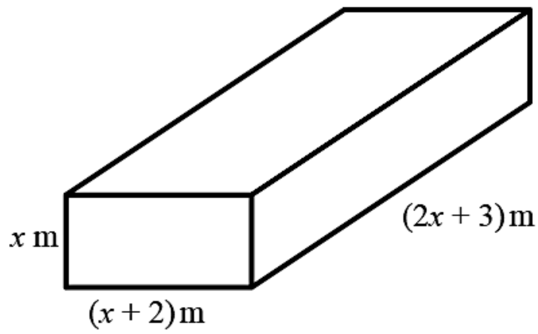
16 The first term of an arithmetic sequence is 100 and the common difference is −2. The sum of the first n terms is 2440. Find the possible values of n.

17 A tank in the shape of a cuboid has the dimensions shown in the diagram.

a Write down an expression for the volume of the tank, V, in terms of x, expanding and simplifying your answer.

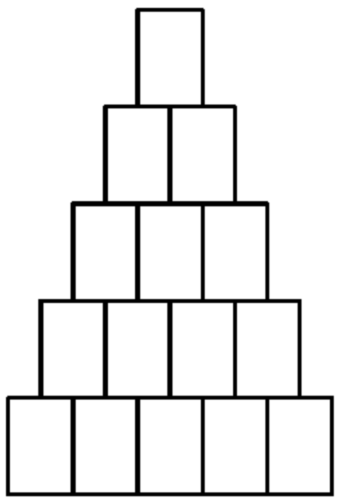
The volume of the tank is $31.5\,\text{m}^3$.

b Find the dimensions of the tank.



18 Clara organizes cans in triangular piles, where each row has one less can than the row below. For example, the pile of 15 cans shown has 5 cans in the bottom row and 4 cans in the row above it.

a A pile has 20 cans in the bottom row. Show that the pile contains 210 cans.
b There are 3240 cans in a pile. How many cans are in the bottom row?
c **i** There are S cans and they are organized in a triangular pile with n cans in the bottom row. Show that $n^2 + n - 2S = 0$.
ii Clara has 2100 cans. Explain why she cannot organize them in a triangular pile.



19 In a class of 30, everybody learns either French or Spanish or both. The number of people who learn Spanish is one more than the number who learn French. The number who learn both is 12 less than the number who only learn one language. A student is picked at random. What is the probability that they learn both French and Spanish?

20 The curve $y = ax^2 + bx$ passes through the point (1, 7) with gradient 12. Find the value of y when $x = 2$.

21 The numerical values of the volume of a cube and its surface area are added together. The answer is 100. Find the length of one edge of the cube, giving your answer to three significant figures.

22 Two numbers differ by one. Their cubes differ by two. Find the possible values of the smaller number.

23 Solve the following simulataneous equations:
$x^3 + y^3 + z^3 = 8$
$x^3 + 2y^3 + 3z^3 = 23$
$x^3 - y^3 + 2z^3 = 18$

24 n people meet and everybody shakes everyone else's hand. There are a total of 190 handshakes. Find the value of n.

Answer:

1 4 cm

2 b $x = -3.19, 4.69$

3 b $x = -3.70, -3, 2.70$

4 a $x^2 + x - 10 = 0$ b 2.70, −3.70

5 1.65 cm

6 a $8s + 6f = 142$; $s + f = 20$
b 11

7 $2

8 39

9 a $x + y = 10\,000$ b 39 AUD
c $12x + 5y = 108\,800$ d $x = 8400, y = 1600$

10 $u_1 = -11, d = 3$

11 a $A = x^2 + 2x$
b $x = 9.5$
c 42 m

12 8

13 0.5 or −1.5

14 −3

15 b $x = 2.73$

16 40 or 61

17 a $V = 2x^3 + 7x^2 + 6x$ b $1.5 \times 3.5 \times 6$

18 b 80
c ii Solutions to $n^2 + n - 4200$ are not whole numbers.

19 0.3

20 24

21 3.28

22 0.264 or −1.26

23 $x = 1, y = -1, z = 2$

24 20

1 Given that $x = 12.475$, express the value of x to:

a one decimal place

b two significant figures.

2 Find the maximum percentage error in the area of a square if the side length is measured to be 0.06 m to two decimal places.

3 Find the percentage error when $\sqrt{2}$ is approximated to one decimal place.

4 Amit invests £60 000 at a fixed annual rate of 3% (compounded annually). His investment will pay an annuity. What is the maximum annual annuity he can draw if he wants to ensure there is at least £10 000 remaining after 25 years of drawing the annuity?

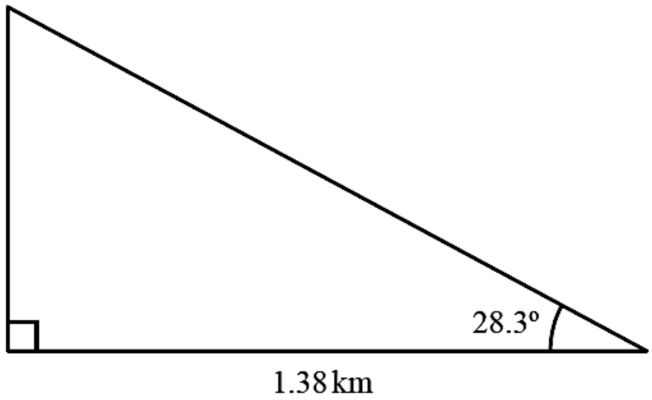
5 José stands 1.38 kilometres from a vertical cliff.

a Express this distance in metres.

José estimates the angle between the horizontal and the top of the cliff as 28.3° and uses it to find the height of the cliff.

b Find the height of the cliff according to José's calculation. **Express your answer in metres, to the nearest whole metre**.

c The actual height of the cliff is 718 metres. Calculate the percentage error made by José when calculating the height of the cliff.



6 Given $p = x - \frac{\sqrt{y}}{z}$, $x = 1.775$, $y = 1.44$ and $z = 48$,

a calculate the value of p.

Barry **first** writes x, y and z correct to one significant figure and **then** uses these values to estimate the value of p.

b **i** Write down x, y and z each correct to one significant figure.

ii Write down Barry's estimate of the value of p.

c Calculate the percentage error in Barry's estimate of the value of p.

7 $T = \frac{(\tan(2z)+1)(2\cos(z)-1)}{y^2 - x^2}$, where $x = 9$, $y = 41$ and $z = 30°$.

a Calculate the **exact** value of T.

b Give your answer to T correct to

i two significant figures;

ii three decimal places.

Pyotr estimates the value of T to be 0.002.

c Calculate the percentage error in Pyotr's estimate.

8 On the right is an extract from a questionnaire response used to survey people attending a doctor's surgery. Use estimation to determine which of the answers should be discarded.

Name:	John Sullie
Age:	27/11/1979
Height in metres:	5.11
Weight in kg:	75
Distance travelled in km:	1.4
Method of travel:	Car

9 Sam measures the circumference of a circle to be 15.1 cm to one decimal place and the diameter of the circle to be 4.8 cm to one decimal place. She uses these measurements to estimate π.

a Find, to four decimal places, the upper and lower bounds of her estimate.

b Find the percentage error if she uses her central values to estimate π.

c Find the largest possible percentage error if she quotes a range as her estimate.

10 A ball is thrown vertically downwards with speed $u = 2.1\,\mathrm{m\,s^{-1}}$ to two significant figures. Its speed t seconds later is measured to be $v = 16\,\mathrm{m\,s^{-1}}$ to two significant figures. Its acceleration is known to be $a = 9.81\,\mathrm{m\,s^{-2}}$ to three significant figures.

Using the formula $v = u + at$, find the upper and lower bounds of t to four significant figures.

11 A rectangular wall has dimensions 3.4 m by 5.2 m, each to two significant figures. A can of paint says it contains 5.00 litres to two decimal places. 1 metre squared of wall requires 2.4 litres to two significant figures.

Joseph wants to buy enough paint to guarantee covering the wall. How many cans should he buy?

12 The value p is given by the inequality $8 \leqslant p < 10$. The value q is given by the inequality $3 \leqslant q < 7$.

Find the maximum percentage error if the central values are used to calculate:

a $p + q$
b $p - q$

13 Yousef uses a model to predict the probability of a popcorn kernel popping (p) based on the temperature (°C). The model claims that:

$$p = e^{\frac{T}{100}} - 1.5$$

a Evaluate the predicted probability if T is:
  i 0
  ii 50
  iii 100.

b Which of the predictions made in **a** are obviously not applicable? Justify your answer.

14 Lianne invests \$1000 at 5% interest for 10 years, compounded monthly.

a She estimates that she will get \$50 interest each year, so a total of \$500 in interest. What is the percentage error in her estimate?

b How much could Lianne withdraw each month if she wants to have \$500 remaining at the end of the ten-year period?

15 Kanmin takes out a loan of ¥24 000. The interest rate for the first year is 2%, compounded monthly. The subsequent annual interest rate is 8%, compounded monthly. If Kanmin makes repayments of ¥300 each month find:

a the number of months required
b the final month's payment.

16 Orlaigh wants to borrow £200 000. She has two options:

Option A: Pay 5% interest rate for 25 years, compounded quarterly.
Option B: Pay 4% interest rate for 30 years, compounded monthly.

Orlaigh will make the minimum required repayment in each compounding period.

Which option should Orlaigh choose if she wishes to minimize the total amount repaid? Justify your answer.

17 Chad takes out a loan of \$100 000 over 20 years. The interest rate is 4.8% compounded monthly.

a How much will Chad pay each month?
b If Chad overpays by \$100 each month, how much interest will he save in total? Give your answer to the nearest \$100.

18 Quinn takes out a loan of €6000 over 10 years at an annual rate of 6%, compounded monthly.

a What is the monthly repayment?
b What percentage of repayments are on interest?
c What percentage of the interest paid is paid in the first five years?

19 If $y = -5$ and $x = 20$, both given to one significant figure, find the maximum value of $(x - y)(x + y)$, giving your answer to two decimal places.

20 It is known that $a = 2.6$ to one decimal place.

- **a** Write down an inequality to describe the possible values of a.
- **b** Write down an inequality to describe the possible values of 10^a to one decimal place.
- **c** Hence give 10^a to a suitable degree of accuracy, justifying your answer.

Answer:

1 a 12.5 b 12

2 19.09%

3 1.01%

4 £3719.95

5 a 1380 m b 743 m c 3.49%

6 a 1.75
b i $x = 2, y = 1, z = 50$
ii 1.98
c 13.1%

7 a 0.001 25
b i 0.0013 ii 0.001
c 60%

8 Age and height in metres

9 a $3.1031 < \pi < 3.1895$
b 0.135%
c 1.52%

10 $1.360 < t < 1.474$

11 9

12 a 27.2% b 300%

13 a i −0.5
ii 0.149
iii 1.22
b i and iii are wrong. Probabilities must be between 0 and 1 inclusive

14 a 22.7% b \$7.39

15 a 99 months b ¥202.81

16 Option B: £343 739 vs £351 486

17 a \$648.96 b \$12 500

18 a €66.61 b 33.2% c 72.4%

19 604.75

20 a $2.55 \leqslant a < 2.65$
b $354.8 \leqslant 10^a < 446.7$
c 400 to one significant figure (maximum significance that includes all possible values)

1 The percentage of charge in a laptop battery, C%, after t minutes of charging can be modelled by the function $C = 1.2t + 16$.

a What is the percentage of charge in the battery when charging begins?

b What percentage is the battery at after half an hour?

c How long does it take to fully charge the battery?

2 The graph of the quadratic function $f(x) = ax^2 + bx + c$ intersects the y-axis at the point (0, 3) and has its vertex at the point (6, 15).

a Write down the value of c.

b By using the coordinates of the vertex, or otherwise, write down two equations in a and b.

c Find the values of a and b.

3 The function $f(x) = p \times 0.4^x + q$ is such that $f(0) = 5$ and $f(1) = 4.1$.

a Find the values of p and q.

b State the equation of the horizontal asymptote of the graph of $y = f(x)$.

4 Charles' Law states that at constant pressure the volume of a gas, V cm^3, is proportional to its absolute temperature, T kelvins. When the temperature of a gas is 350 kelvins its volume is 140 cm^3.

a Find the relationship between volume and absolute temperature for the gas.

b Given that 0 kelvin = –273 °C, find the volume of the gas at room temperature of 20 °C.

5 The number of apartments in a housing development has been increasing by a constant amount every year. At the end of the first year the number of apartments was 150, and at the end of the sixth year the number of apartments was 600. The number of apartments, y, can be determined by the equation $y = mt + n$, where t is the time, in years.

a Find the value of m.

b State what m represents **in this context**.

c Find the value of n.

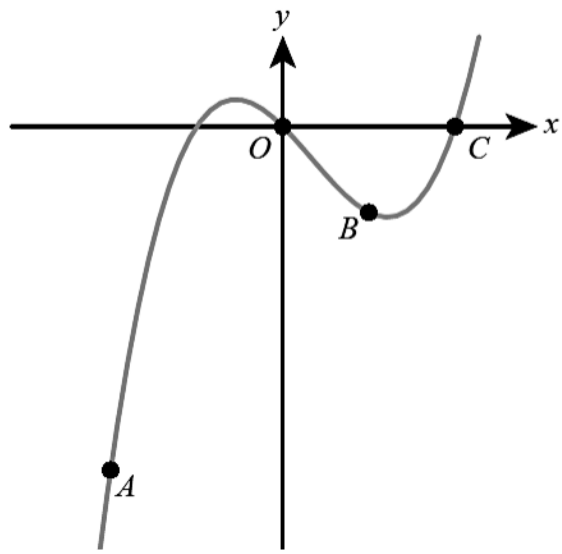
d State what n represents **in this context**.

6 Consider the function $f(x) = px^3 + qx^2 + rx$. Part of the graph of f is shown opposite.

The graph passes through the origin O and the points $A(-2, -8)$, $B(1, -2)$ and $C(2, 0)$.

a Find three linear equations in p, q and r.

b Hence find the value of p, of q and of r.



7 The diagram shows the graph of a cosine function, $g(x) = a \cos bx + c$ for $-360° \leqslant x \leqslant 360°$, and a sine function, f(x).

a Write down

i the amplitude of f(x)

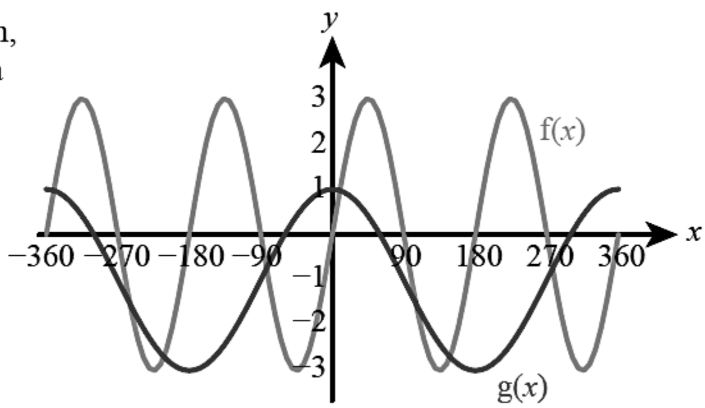
ii the period of f(x).

b Write down the value of

i a

ii b

iii c.



c Write down the number of solutions of $f(x) = g(x)$ in the domain $-180° \leqslant x \leqslant 360°$.

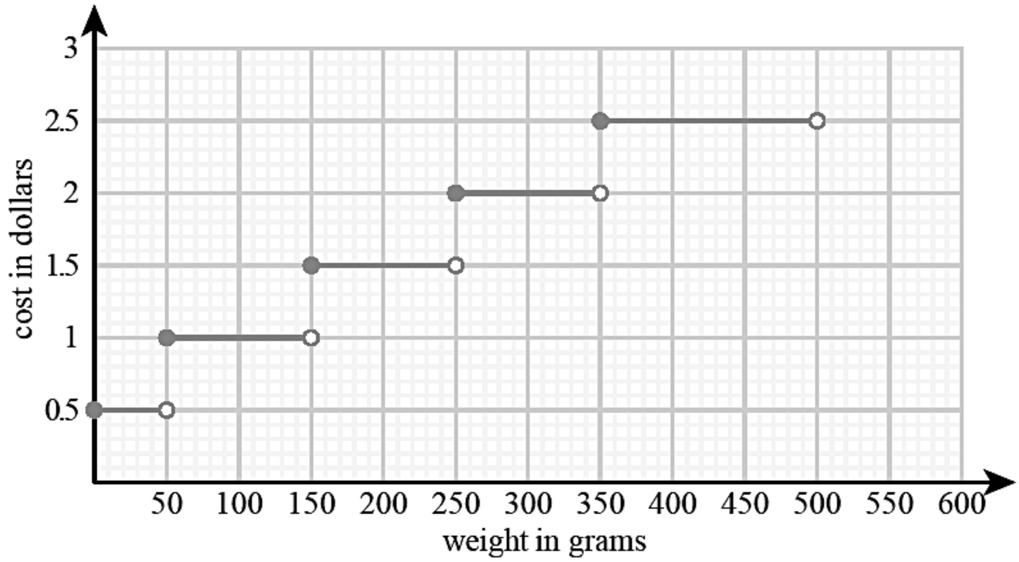
8 The graph shows the cost, in dollars, of posting letters with different weights.

a Write down the cost of posting a letter weighing 60 g.

b Write down the cost of posting a letter weighing 250 g.

Kathy pays 2.50 dollars to post a letter.

c Write down the range for the weight, w, of the letter.

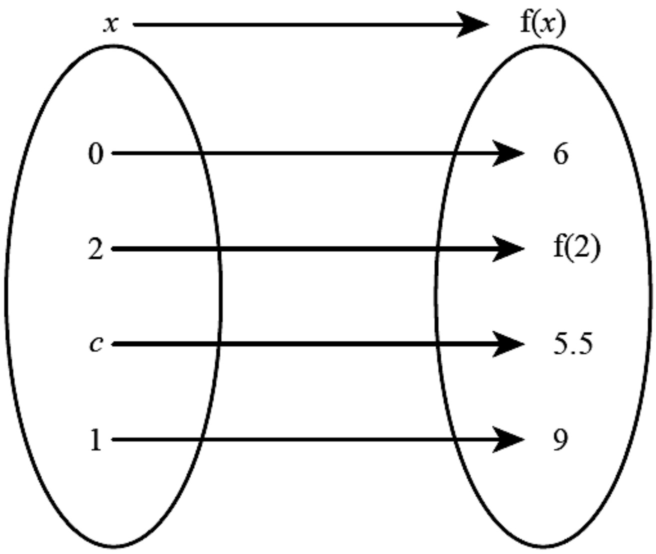


9 The function $f(x) = a^x + b$ is defined by the mapping diagram opposite.

a Find the values of a and b.

b Write down the image of 2 under the function f.

c Find the value of c.



10 The gravitational force on an object is inversely proportional to the square of the distance of the object from the centre of the Earth. A satellite is launched into orbit 1500 km above the Earth's surface.

Given that the radius of the Earth is 6000 km, find the percentage decrease in the gravitational force on the satellite when launched into orbit.

11 George leaves a cup of hot coffee to cool and measures its temperature every minute. His results are shown in the table below.

Time, *t* (minutes)	0	1	2	3	4	5	6
Temperature, *y* (°C)	94	54	34	24	k	16.5	15.25

a Write down the decrease in the temperature of the coffee

i during the first minute (between $t = 0$ and $t = 1$)

ii during the second minute

iii during the third minute.

b Assuming the pattern in the answers to part **a** continues, show that $k = 19$.

c Use the **seven** results in the table to draw a graph that shows how the temperature of the coffee changes during the first six minutes.
Use a scale of 2 cm to represent 1 minute on the horizontal axis and 1 cm to represent 10 °C on the vertical axis.

The function that models the change in temperature of the coffee is $y = p(2^{-t}) + q$.

d **i** Use the values $t = 0$ and $y = 94$ to form an equation in p and q.
ii Use the values $t = 1$ and $y = 54$ to form a second equation in p and q.

e Solve the equations found in part **d** to find the value of p and the value of q.

The graph of this function has a horizontal asymptote.

f Write down the equation of this asymptote.

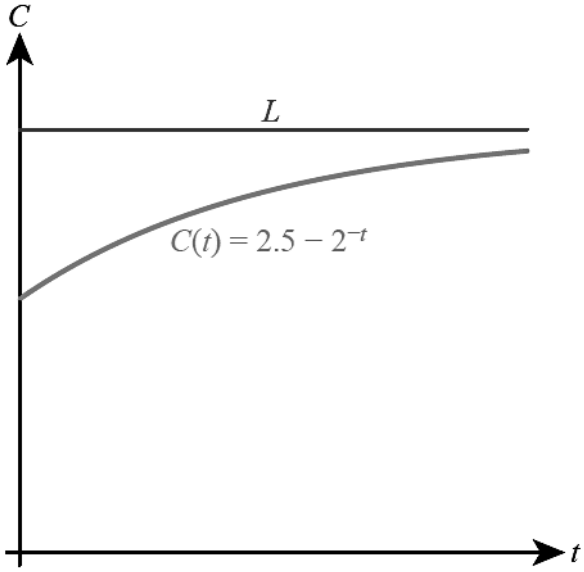
George decides to model the change in temperature of the coffee with a linear function using correlation and linear regression.

g Use the **seven** results in the table to write down
i the correlation coefficient
ii the equation of the regression line y on t.

h Use the equation of the regression line to estimate the temperature of the coffee at $t = 3$.

i Find the percentage error in this estimate of the temperature of the coffee at $t = 3$.

12 The amount of electrical charge, C, stored in a mobile phone battery is modelled by $C(t) = 2.5 - 2^{-t}$, where t, in hours, is the time for which the battery is being charged.



a Write down the amount of electrical charge in the battery at $t = 0$.

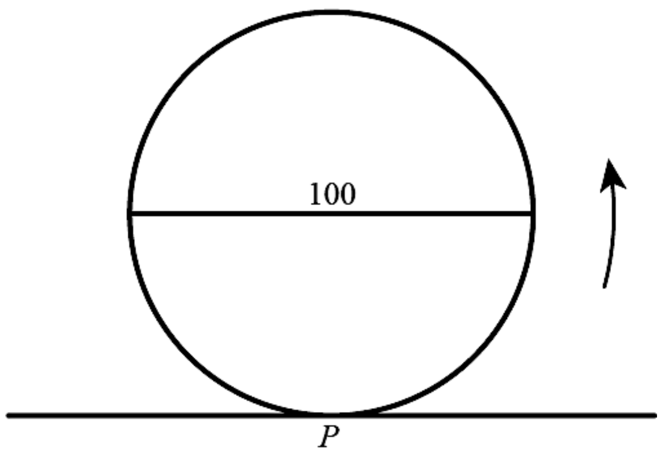
The line L is the horizontal asymptote to the graph.

b Write down the equation of L.

To download a game to the mobile phone, an electrical charge of 2.4 units is needed.

c Find the time taken to reach this charge.
Give your answer correct to the nearest minute.

13 The diagram opposite represents a big wheel, with a diameter of 100 metres.



Let P be a point on the wheel. The wheel starts with P at the lowest point, at ground level. The wheel rotates at a constant rate, in an anticlockwise (counterclockwise) direction. One revolution takes 20 minutes.

a Write down the height of P above ground level after
i 10 minutes
ii 15 minutes.

Let $h(t)$ metres be the height of P above ground level after t minutes. Some values of $h(t)$ are given in the table below.

t	h(*t*)
0	0.0
1	2.4
2	9.5
3	20.6
4	34.5
5	50.0

b **i** Show that h(8) = 90.5.

ii Find h(21).

c **Sketch** the graph of h, for $0 \leqslant t \leqslant 40$.

d Given that h can be expressed in the form $h(t) = a \cos bt + c$, find a, b and c.

Answer:

1 a 16% b 52%

c 70 minutes

2 a $c = 3$ b $6 = -\frac{b}{2a}$

$12 = 36a + 6b$

c $a = -\frac{1}{3}$, $b = 4$

3 a $p = 1.5$, $q = 3.5$ b $y = 3.5$

4 a $V = 0.4T$ b $V = 117\,\text{cm}^3$

5 a $m = 90$

b Change per year in the number of apartments

c 60

d Number of apartments initially

6 a $-8p + 4q - 2r = -8$

$p + q + r = -2$

$8p + 4q + 2r = 0$

b $p = 1$, $q = -1$, $r = -2$

7 a i 3 ii 180°

b i $a = 2$ ii $b = 1$ iii $c = -1$

c 5

8 a \$1.00 b \$2.00

c $350 \leqslant w < 500$

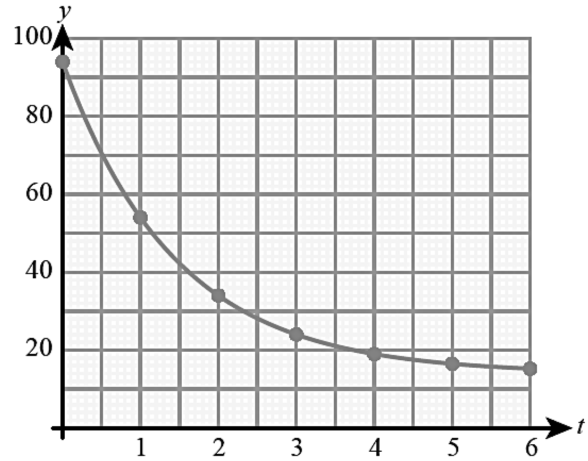
9 a $a = 4$, $b = 5$ b 2

c $c = -\frac{1}{2}$

10 36%

11 a i 40 °C ii 20 °C iii 10 °C

c



d i $94 = p + q$ ii $54 = \frac{p}{2} + q$

e $p = 80$, $q = 14$ f $y = 14$

g i $r = -0.878$ ii $y = 71.6 - 11.7t$

h 36.7 i 52.8%

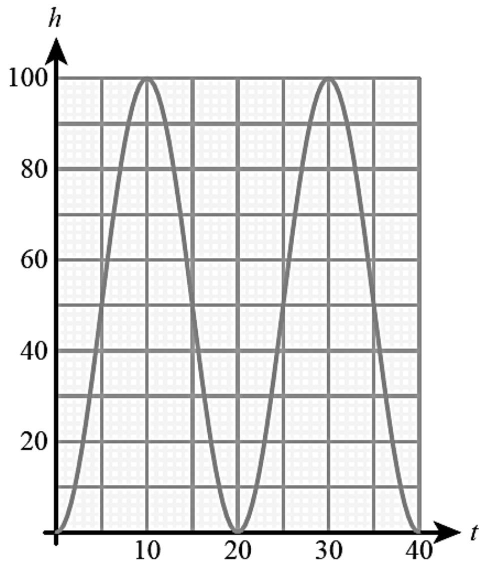
12 a 1.5 b $C = 2.5$

c 3 hours 19 minutes

13 a i 100 m ii 50 m

b ii 2.4

c



d $a = -50$, $b = 18$, $c = 50$

1 **a** Find the coordinates of the point A on the curve with equation $y = 3x^2 - 4x$ where the tangent is horizontal.

b Sketch the curve and show the position of point A.

2 The graph of $y = 4x^2 - bx$ has a horizontal tangent when $x = -2$. Find the value of b.

3 A function is given by $f(x) = 4x^3 - 5x$.

a Sketch the graph of $y = f'(x)$.

b Hence solve the equation $f'(x) = 0$.

4 Given that $y = 4x^2 - \frac{5}{x}$, use your graphical calculator to solve the equation $\frac{dy}{dx} = 0$.

5 Find the coordinates of the local minimum point on the graph with equation $y = 4x^3 - 3x + 8$.

6 **a** Sketch the graph of $y = 9x^2 - x^4$ for $0 \leqslant x \leqslant 3$, showing the coordinates of the maximum and minimum points.

b Use the trapezoidal rule with six strips to estimate the area between the graph and the x-axis.

7 Use the trapezoidal rule with five strips to find an approximate value of $\int_2^{12} \ln\left(\frac{x}{2}\right) dx$.

8 A rectangle with sides w cm and h cm has perimeter 88 cm.

a Express h in terms of w and hence show that the area of the rectangle is given by $A = 44w - w^2$.

b Find the values of w and h for which the rectangle has the maximum possible area.

c Find the maximum possible area of the rectangle.

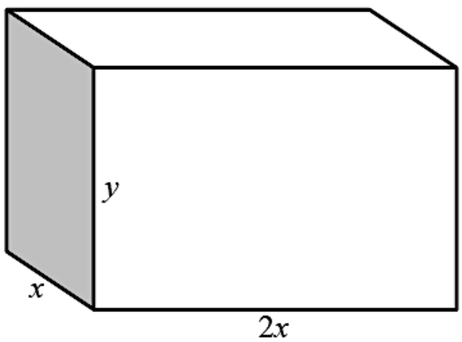
9 The fuel consumption of a car, L litres per 100 km, varies with speed, v km h^{-1} according to the model model $L = 17.3 + 0.03(x - 70) + 0.0001(x - 70)^4$. At what speed should the car be driven in order to minimize fuel consumption?

10 A shipping container is to be made with six rectangular faces, as shown in the diagram.

The dimensions of the container are
length $2x$
width x
height y.

All of the measurements are in metres.

The total length of all 12 edges is 48 metres.



a Show that $y = 12 - 3x$.

b Show that the volume V m^3 of the container is given by $V = 24x^2 - 6x^3$.

c Find $\frac{dV}{dx}$.

d Find the value of x for which V is a maximum.

e Find the maximum volume of the container.

f Find the length and height of the container for which the volume is a maximum.

The shipping container is to be painted. One litre of paint covers an area of 15 m^2. Paint comes in tins containing four litres.

g Calculate the number of tins required to paint the shipping container.

11 Find the coordinates of the point on the graph of $y = x^2 - \frac{3}{x}$ where the gradient is zero.

12 The curve with equation $y = ax^2 - 48x + b$ has a horizontal tangent at (8, 21). Find the values of a and b.

13 The graph of $y = 3x^2 - \frac{a}{x}$ has a horizontal tangent when $x = -1$. Find the coordinates of the local minimum point on the graph.

14 **a** Use the trapezoidal rule with four intervals to estimate the value of $\int_0^4 e^{-x}\, dx$.

b Use your calculator to evaluate the integral and hence find the percentage error in your estimate.

15 A function is defined on the domain $0 \leqslant x \leqslant 3$ by $f(x) = x^3 - 3x^2 + 2x$.

a Find the coordinates of the local maximum point of the function.

b Find the greatest value of the function on this domain.

16 Find the minimum value of $0.2x^4 - 3x^3 + 7.5x^2 + 1.3x + 1$ for $0 \leqslant x \leqslant 10$.

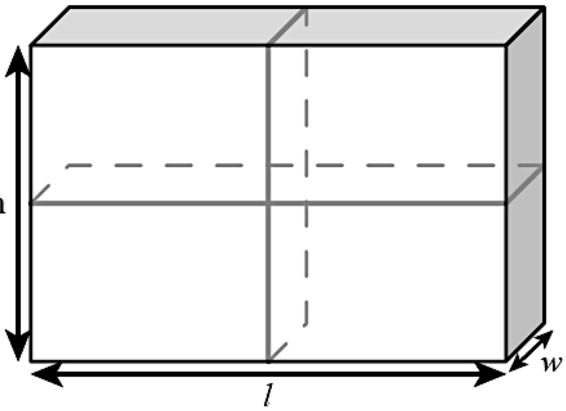
17 Metal bars can be produced either in the shape of a square based cuboid or a cylinder. The volume of the bars is fixed at $300\,\text{cm}^3$. Find the shape and the dimensions of the bar with the smallest possible surface area.

18 A parcel is in the shape of a rectangular prism, as shown in the diagram. It has a length l cm, width w cm and height of 20 cm. The total volume of the parcel is $3000\,\text{cm}^3$.

a Express the volume of the parcel in terms of l and w.

b Show that $l = \frac{150}{w}$.

The parcel is tied up using a length of string that fits **exactly** around the parcel, as shown in the diagram.



c Show that the length of string, S cm, required to tie up the parcel can be written as

$$S = 40 + 4w + \frac{300}{w}, \ 0 < w \leqslant 20.$$

d Draw the graph of S for $0 < w \leqslant 20$ and $0 < S \leqslant 500$, clearly showing the local minimum point. Use a scale of 2 cm to represent 5 units on the horizontal axis w (cm), and a scale of 2 cm to represent 100 units on the vertical axis S (cm).

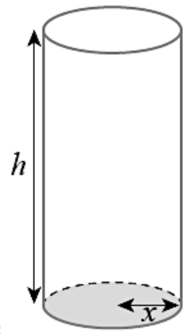
e Find $\frac{dS}{dw}$.

f Find the value of w for which S is a minimum.

g Write down the value, l, of the parcel for which the length of string is a minimum.

h Find the minimum length of string required to tie up the parcel.

19 A dog food manufacturer has to cut production costs. They wish to use as little aluminium as possible in the construction of cylindrical cans. In the following diagram, h represents the height of the can in cm and x, the radius of the base of the can in cm. The volume of the dog food cans is $600\,\text{cm}^3$.



a Show that $h = \frac{600}{\pi x^2}$.

b **i** Find an expression for the curved surface area of the can, in terms of x. Simplify your answer.

ii Hence write down an expression for A, the total surface area of the can, in terms of x.

c Differentiate A in terms of x.

d Find the value of x that makes A a minimum.

e Calculate the minimum total surface area of the dog food can.

Mathematical Studies SL May 2010 TZ2 Paper 2 Q5

20 The graph of $y = ax^3 + 2x^2 + bx + 45$ has a horizontal tangent at $(2, 5)$.
Find the second point on the graph where the tangent is horizontal.

21 The temperature in an office is monitored over a period of 24 hours. It is found that the temperature, T °C, can be modelled as a cubic function of t, the time in hours after midnight. The minimum temperature of 4 °C was recorded at 2 am. The maximum temperature of 28 °C was recorded at 3 pm.

a Find the cubic model for how the temperature changes with time. Give the coefficients to two significant figures.

b What temperature does this model predict for 11.50 pm? State one limitation of this model.

22 a is 20 to one significant figure. b is 22 to two significant figures.

The function f is defined by $f(x) = (x - 22)(26 - x)$.

Find the largest value of $f(a) - f(b)$.

23 A field is represented on a map. Several points around the edge of the field are marked and their coordinates noted:

(7, 8), (8, 6), (9, 4), (10, 3), (11, 3), (12, 3),
(12, 13), (11, 12), (10, 12), (9, 11), (8, 9)

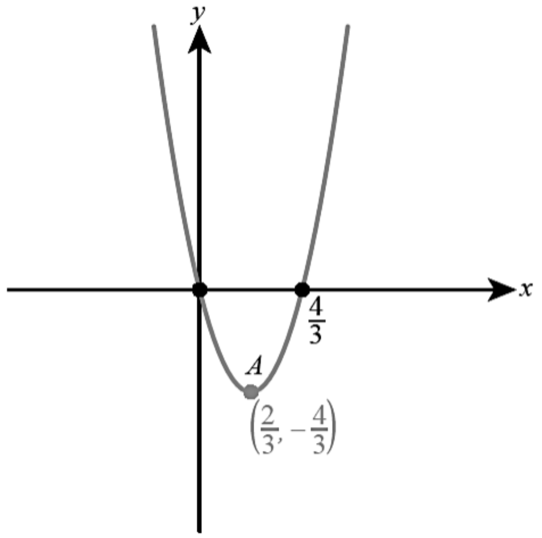
a Use these coordinates to estimate the area of the field on the map.

b The units on the map are centimetres, and the map scale is 1 unit : 25 m. Find an estimate of the actual area of the field.

Answer:

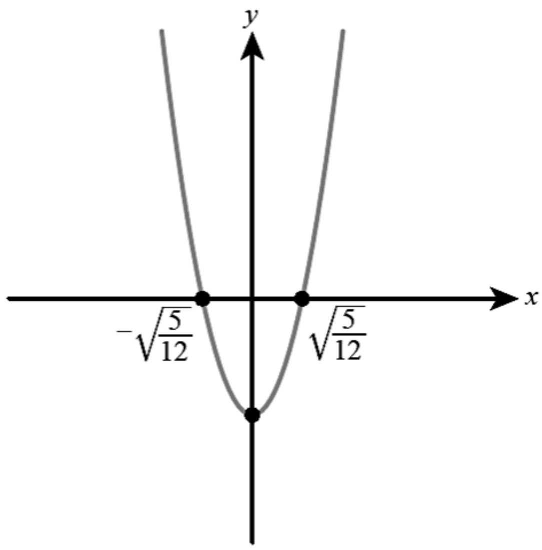
1 a $\left(\frac{2}{3}, -\frac{4}{3}\right)$

b



2 −16

3 a

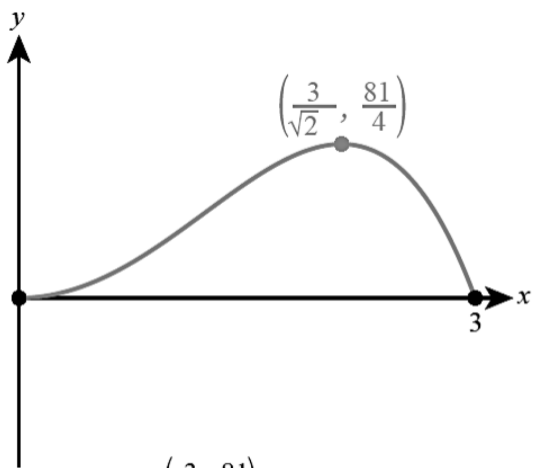


b $\pm\sqrt{\frac{5}{12}}$

4 −0.855

5 (0.5, 7)

6 a



maximum at $\left(\frac{3}{\sqrt{2}}, \frac{81}{4}\right)$, mimima in the interval at the end points (0, 0) and (3, 0)

b 31.3

7 11.4

8 a $h = 44 - w$

b $w = 22$, $h = 22$

c $484\,\text{cm}^2$

9 $65.8\,\text{km}\,\text{h}^{-1}$

10 c $48x - 18x^2$

d 2.67 m

e $56.9\,\text{m}^3$

f 5.33 m, 4 m

g 7

11 (−1.14, 3.93)

12 $a = 3$, $b = 213$

13 (−1, 9)

14 a 1.06

b 0.982, 8.20%

15 a (0.423, 0.385)

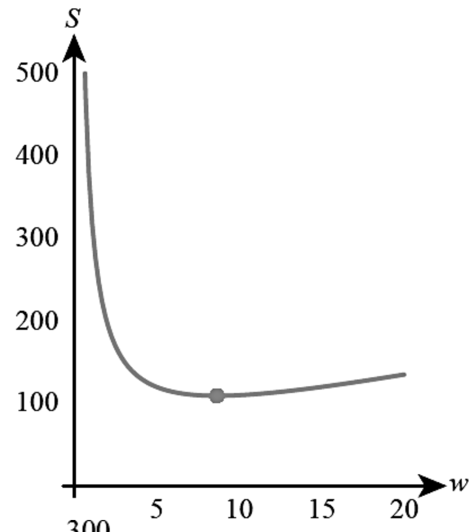
b 6

16 −256

17 cylinder, $r = 3.63$ cm, $h = 7.26$ cm

18 a $V = 20lw$

d



e $4 - \frac{300}{w^2}$

f 8.66

g 17.3

h 109 cm

19 b i $\frac{1200}{x}$

ii $A = 2\pi x^2 + \frac{1200}{x}$

c $\frac{dA}{dx} = 4\pi x - \frac{1200}{x^2}$

d 4.57

e $394\,\text{cm}^2$

20 (−2.67, 107)

21 a $T = -0.022t^3 + 0.56t^2 - 2.0t + 5.9$

b −21.5 °C; it predicts a much lower temperature at midnight on the next day than the previous day

22 6.25

23 a $33\,\text{units}^2$

b $20\,625\,\text{m}^2$

1 The following data shows the amount of oxygen produced (V ml) by a plant in an hour at different levels of carbon dioxide (c%).

c	0.01	0.02	0.03	0.04	0.05	0.06	0.07
V	5.1	10.3	14.8	19.6	20.8	22.3	22.4

a Find the Pearson's product-moment correlation coefficient of this data
b Find the Spearman's rank correlation coefficient of this data.
c What do your answers to **a** and **b** tell you about this data?

2 Juan plays a quiz game. The scores he achieves on the separate topics may be modelled by independent normal distributions.
a On the topic of sport, the scores have the distribution $N(75, 12^2)$.
Find the probability that Juan scores less than 57 points on the topic of sport.
b Juan claims that he scores better in current affairs than in sport. He achieves the following scores on current affairs in 10 separate quizzes.
91 84 75 92 88 71 83 90 85 78
Perform a hypothesis test at the 5% significance level to decide whether there is evidence to support his claim.

3 A six-sided dice is thrown 300 times and the outcomes recorded in the following table.

Score	1	2	3	4	5	6
Frequency	45	57	51	56	47	44

Perform a suitable test at the 5% level to determine if the dice is fair.

4 A six-sided dice is rolled 300 times and the following results are recorded:

Outcome	1	2	3	4	5	6
Frequency	42	38	55	61	46	58

A test is conducted to check whether the dice is biased.
a State suitable null and alternative hypotheses.
b Find the expected frequencies.
c State the number of degrees of freedom.
d State the conclusion of the test at the 10% level of significance and justify your answer.

5 Su-Yong is investigating the relationship between hours of sunshine per day and average daily temperature (in °C) in a number of different locations. She collects the following data:

Location	A	B	C	D	E	F	G
Hours of sunshine	7.3	8.7	8.1	5.9	4.3	7.2	6.5
Average temperature	11.4	18.2	12.7	9.8	10.0	17.5	11.2

a Calculate the Spearman's rank correlation coefficient for this data.
b Su-Yong thinks that there is a positive correlation between the hours of sunshine and temperature. Write down suitable hypotheses she could use to test this.
c The critical value for this test is 0.571. What should Su-Yong conclude?

6 A train company claims that times for a particular journey are normally distributed with mean 17.5 minutes. Lenka makes this journey to school every morning and thinks that they take longer on average. She records the times taken on eight randomly selected days, correct to the nearest half-minute:

7 A fruit grower knows that the average yield of his apple trees is 16 bushels. He decides to stop using artificial fertilizers and wants to test whether this has led to decreased yield. The following year, the yields from 10 randomly selected trees had mean 14.7 bushels and the estimate of population standard deviation was 2.3 bushels. He conducts a t-test, using a 5% level of significance, to test whether there has been a decrease in average yield.

a State suitable hypotheses for this test.

b State the conclusion the fruit grower should draw.

8 A teacher works at two different schools and wants to find out whether the grades his students get depend on the school. She has the following data:

	3	4 or 5	6 or 7
School A	11	44	36
School B	16	83	40

The teachers decides to conduct a χ^2 test for independence, using the 5% level of significance.

a Write down suitable hypotheses for this test.

b Calculate the p-value and hence state the conclusion.

9 Francisco and his friends want to test whether performance in running 400 metres improves if they follow a particular training schedule. The competitors are tested before and after the training schedule.

The times taken to run 400 metres, in seconds, before and after training are shown in the following table.

Competitor	A	B	C	D	E
Time before training	75	74	60	69	69
Time after training	73	69	55	72	65

Apply an appropriate test at the 1% significance level to decide whether the training schedule improves competitors' times, stating clearly the null and alternative hypotheses. (It may be assumed that the distributions of the times before and after training are normal.)

10 A toy manufacturer makes a cubical dice with the numbers 1, 2, 3, 4, 5, 6 respectively marked on the six faces. The manufacturer claims that, when it is thrown, the probability distribution of the score X obtained is given by

$P(X = x) = \frac{x}{21}$ for $x = 1, 2, 3, 4, 5, 6$.

To check this claim, Pierre throws the dice 420 times with the following results.

x	**Frequency**
1	25
2	46
3	64
4	82
5	99
6	104

State suitable hypotheses and, using an appropriate test, determine whether or not the manufacturer's claim can be accepted at the 5% significance level.

11 A baker claims that his loaves of bread weigh 800 g on average. A customer believes that the average weight is less than this. A random sample of ten loaves is weighed, with the following results (in grams):
803, 785, 780, 800, 801, 791, 783, 781, 807, 783

a Find the mean of the sample.

In spite of these results, the baker still insists that his claim is correct.

b Test this claim at the 10% significance level, stating your hypotheses and conclusion clearly. You may assume that the data comes from a normal distribution.

12 Hilary keeps two chickens. She wants to determine whether the eggs laid by the two chickens have the same average mass. She weighs the next ten eggs laid by each chicken.

a State a possible problem with this sampling technique.

The masses of the ten eggs (in grams) are:

Chicken A	53.1	52.7	56.3	51.2	53.7	50.9	51.2	55.1	52.8	52.0
Chicken B	56.3	54.1	52.7	50.2	51.4	55.0	50.9	51.3	54.9	56.2

b Write down suitable hypotheses.

c State two assumptions you need to make in order to use a t-test.

d Conduct the test at the 5% significance level and state your conclusion clearly.

13 The scores on a Maths test and a Geography test for a group of ten students are as follows.

Student	A	B	C	D	E	G	H	I	J	K
Maths	34	36	44	49	50	58	59	60	60	60
Geography	34	33	35	39	25	40	32	32	28	26

a Calculate the Spearman's rank correlation coefficient between the two sets of data.

A teacher wants to test whether there is any correlation between the two sets of scores.

b Write down suitable null and alternative hypotheses.

c Using the 5% level of significance, the critical value for a one-tailed test is 0.418 and for a two-tailed test is it 0.564. What should the teacher conclude? Justify your answer.

14 A manufacturer of batteries wants to test whether there is any difference in average lifetime of two types of battery. They test a sample of batteries, obtaining the following results (lifetime in hours).

	Sample size	**Sample mean**	**Estimate of population variance**
Type A	18	1286	110,403
Type B	11	1064	130,825

a Use a suitable test, with a 10% significance level, to determine whether there is any difference in mean lifetimes.

b State two assumptions you need to make in order to use your test.

15 Roy is practising archery. He thinks that he has the probability 0.45 of hitting the target.
To test this, he takes five shots at a time and records how many times he hit the target.
He repeats this 100 times, obtaining the following results.

Outcome	0	1	2	3	4 or 5
Observed	12	18	34	22	14

a Assuming Roy is correct, write down the distribution of the number of hits out of 5 shots.
b Hence find the expected frequencies.
c Write down the number of degrees of freedom for a χ^2 goodness of fit test.
d Test using the 5% level of significance whether Roy's belief is correct.

16 Ana tosses seven coins and counts the number of tails. She repeats the experiment 800 times and uses the results to test whether the coins are fair. The results are shown in the following table.

Number of tails	0	1	2	3	4	5	6	7
Frequency	12	34	151	218	223	126	32	4

a State suitable hypotheses for a χ^2 test.
b Calculate expected frequencies, giving your results to two decimal places.
c Find the p-value and the value of the χ^2 statistic.
d State the conclusion of the test at the 2% significance level.

17 Collectable cards come in packs of five. There are a number of special 'shiny cards' which are particularly sought after. The manufacturer claims that one-fifth of cards are 'shiny' and they are distributed at random among the packets. Anwar selected a random sample of 200 packs, and found the following results:

Number of shiny cards	0	1	2	3, 4 or 5
Frequency	70	80	40	10

a Use this information to estimate the average number of shiny cards in a pack. What does this tell you about the manufacturer's claim?
b Test the manufacturer's claim at the 5% significance level. State
  i the null and alternative hypotheses
  ii the value of the test statistic
  iii the p-value
  iv the conclusion of the test, in context.

18 Jamie wants to know if there is a tendency for people's 100 m times to decrease with age. He tracks his own best times each year:

Age	100 m time
10	16.2
11	14.8
12	13.9
13	14.1
14	14.4
15	13.6
16	13.4

a Assuming that the differences between times each year follow a normal distribution, is there evidence at the 5% significance level that the average change each year is negative?
b If the difference between times each year does not follow a normal distribution, the same question can be answered using Spearman's rank correlation coefficient.
  i State the null and alternative hypotheses.
  ii Find the value of Spearman's rank correlation coefficient.
  iii Given that the appropriate critical value at 5% significance is 0.714, what is the appropriate conclusion? Give your answer in context.

19 An examiner claims that test scores (out of 100) follow a normal distribution with mean 62 and variance 144. A teacher believes that the mean is lower. She looks at a sample of 80 scores and finds that the sample mean is 60.4 and the estimate of population variance is 144.2.

a Conduct a suitable test to show that, at the 5% level of significance, there is insufficient evidence for the teacher's belief.

A student reminds the teacher that, for her test to be valid, the population distribution of the scores must be normal. The 80 scores in the sample were distributed as follows:

Score (s)	$\leqslant 45$	$45 < s \leqslant 55$	$55 < s \leqslant 65$	$65 < s \leqslant 75$	$s > 75$
Frequency	11	23	22	18	6

b Show that, at the 5% level of significance, there is sufficient evidence that the scores do not come from the distribution N(62, 144).

c The examiner says that the above test shows that the scores are not distributed normally. The teacher says that there are other possible conclusions. State one possible alternative conclusion.

d State another reason, unrelated to the tests above, why a normal distribution may not be a suitable model for the scores.

20 The following table shows the results of a survey into student satisfaction in a school. The students were asked to give a satisfaction score on a scale from 1 to 6:

Score	**1**	**2**	**3**	**4**	**5**	**6**
Frequency	25	30	32	27	34	32

Jane wants to ask various questions about this data, using a 10% significance level. She remembers the following fact:

The critical values of Spearman's rank with this sample size are 0.829 and 0.657 for the one-tailed and two-tailed tests.

But she can't remember which critical value corresponds to which type of test.

Stating any necessary assumptions for each test, determine at the 10% significance level:

a if the mean score has changed from the previous year's average of 3.3

b if there is a tendency for higher scores to be more popular

c if the students were choosing numbers at random (i.e. each score is equally likely).

Answer:

1 a 0.946 b 1

c As c increases V increases, but not in a perfectly linear pattern

2 a 0.0668

b Sufficient evidence to support claim ($p = 0.001\,86$)

3 Insufficient evidence to reject H_0 ($p = 0.681$)

4 a H_0: The dice is unbiased (all outcomes have equal probability), H_1: The dice is biased

b all = 50 c 5

d Insufficient evidence that the dice is biased, $p = 0.123 > 0.10$

5 a 0.857

b H_0: There is no correlation between hours of sunshine and temperature, H_1: There is a positive correlation

c There is sufficient evidence of positive correlation.

6 Sufficient evidence that the mean time is greater than 17.5 minutes ($p = 0.0852 < 0.10$)

7 a $H_0: \mu = 16$, $H_1: \mu < 16$

b Insufficient evidence of decrease ($p = 0.0538 > 0.05$)

8 a H_0: The grades are independent of the school H_1: The grades depend on the school

b $p = 0.198$, insufficient evidence that grades depend on school

9 H_0: The times are the same after training, H_1: The times have decreased after training. Insufficient evidence that times have improved ($p = 0.277$)

10 H_0: The data comes from the given distribution
H_1: The data does not come from this distribution
Insufficient evidence to reject claim ($p = 0.465$)

11 a 791

b H_0: The average weight of a loaf is 800 g, H_1: The average weight of a loaf is less than 800 g. $t = -2.63$, $p = 0.0137$. Sufficient evidence to reject H_0 and say the average weight of a loaf is less than 800 g.

12 a not random (observations may not be independent)

b $H_0: \mu_1 = \mu_2$, $H_1: \mu_1 \neq \mu_2$

c The masses of the eggs of both chickens are distributed normally with equal variance.

d $p = 0.666$, insufficient evidence that the means of egg masses for the two chickens are different

13 a –0.511

b H_0: no correlation, H_1: some correlation

c Insufficient evidence of correlation ($0.511 < 0.564$)

14 a $t = 1.689$, $p = 0.103$, do not reject H_0

b The lifetimes follow a normal distribution with equal variances.

15 a B(5, 0.45)

b 5.03, 20.6, 33.7, 27.6, 13.1

c 4

d $p = 0.0249$, sufficient evidence that Roy's belief is incorrect

16 a H_0: All the coins are fair (P(tails = 0.5))
H_1: (Some of) the coins are biased

b 6.25, 43.75, 131.25, 218.75, 218.75, 131.75, 43.75, 6.25

c $p = 0.0401$, $\chi^2 = 14.7$

d Insufficient evidence that the coins are not fair

17 a Mean is 1, which is consistent with the claimed B(5,0.2) distribution but does not prove it must be that

b i H_0: data is drawn from a B(5,0.2) distribution

H_1: data is not drawn from a B(5,0.2) distribution

ii $\chi^2 = 0.588$

iii $p = 0.899$

iv There is no significant evidence to doubt the manufacturer's claim.

18 a $t = -1.70$, $p = 0.0754$
No: there is insufficient evidence to reject H_0

b i H_0: There is no correlation
H_1: There is negative correlation

ii –0.857

iii Reject H_0 (as age increases there is a tendency for 100 m time to decrease)

19 a $t = -1.19$, $p = 0.118 > 0.05$

b $\chi^2 = 9.79$, $p = 0.0440 < 0.05$

c They could come from a normal distribution with a different mean (or different variance).

d Scores are discrete, normal distribution is continuous.

20 a $t = 2.51$, $p = 0.0129$. Reject H_0 (significant change from 3.3)

b $r_S = 0.696$, do not reject H_0 (no significant evidence of increasing frequency with score)

c $\chi^2 = 1.93$, $p = 0.858$, do not reject H_0 (no significant evidence of non-random pattern)